Chapter 4

Solving Inequalities and Problems

Solving Inequalities

OBJECTIVES for Sections 4-1 through 4-5:
1. *To use the properties of order as reasons for statements in proving theorems.*
2. *To solve inequalities and graph their solution sets.*
3. *To find the intersection and the union of two sets.*
4. *To solve conjunctions and disjunctions.*
5. *To solve open sentences involving absolute value.*

4–1 Properties of Order

One and only one of the following statements is true.

$$2 < -3 \qquad 2 = -3 \qquad -3 < 2$$

The true statement is $-3 < 2$. This situation illustrates the following fact that is used in comparing real numbers.

Axiom of Comparison

For all real numbers a and b, one and only one of the following statements is true:

$$a < b \qquad a = b \qquad b < a$$

Recall from Section 1-1 that the statement $a < b$ gives the same information as the statement $b > a$.

Another fact about order in the set of real numbers can be easily shown on a number line. Suppose you know that a, b, and c are real numbers such that $a < b$ and $b < c$. What is the relationship between a and c? If the graphs of a, b, and c are on a horizontal number line that is marked with positive numbers to the right, the graph of a is to the left of the graph of b, and the graph of b is to the left of the graph of c. Thus you can see that the graph of a is to the left of the graph of c and, therefore, $a < c$.

This example suggests the following property of the real numbers.

Transitive Property of Order

For all real numbers a, b, and c:

1. If $a < b$ and $b < c$, then $a < c$.
2. If $a > b$ and $b > c$, then $a > c$.

What happens when the same number is added to each side of an inequality such as $-4 < 3$?

Add 5: Is it true that $-4 + 5 < 3 + 5$?
Yes, $1 < 8$.
Add -5: Is it true that $-4 + (-5) < 3 + (-5)$?
Yes, $-9 < -2$.

These examples suggest another property of order for the real numbers.

Addition Property of Order

For all real numbers a, b, and c:

1. If $a < b$, then $a + c < b + c$ and $c + a < c + b$.
2. If $a > b$, then $a + c > b + c$ and $c + a > c + b$.

Since subtraction is defined in terms of addition, the addition property of order applies when you subtract a real number from each side of an inequality. For example,

$$\text{if } a < b,$$
$$\text{then } a - 2 < b - 2$$

since $a + (-2) < b + (-2)$.

What happens when each side of the inequality $-4 < 3$ is *multiplied* by a nonzero real number?

Multiply by 5: Is it true that $-4(5) < 3(5)$?
Yes, $-20 < 15$.

Multiply by -5: Is it true that $-4(-5) < 3(-5)$?
No, $20 > -15$.

These examples suggest the following rules.

1. Multiplying each side of an inequality by a positive number *preserves* the order of the inequality.
2. Multiplying each side of an inequality by a negative number *reverses* the order of the inequality.

These rules are stated formally as the following property of order.

Multiplication Property of Order

For all real numbers a, b, and c:

1. If $a < b$ and $c > 0$, then $ac < bc$ and $ca < cb$.
If $a > b$ and $c > 0$, then $ac > bc$ and $ca > cb$.

2. If $a < b$ and $c < 0$, then $ac > bc$ and $ca > cb$.
If $a > b$ and $c < 0$, then $ac < bc$ and $ca < cb$.

Note that multiplying both sides of an inequality by zero does not produce an inequality; the result is the identity $0 = 0$.

Since division is defined in terms of multiplication, the multiplication property of order applies when each side of an inequality is divided by a nonzero real number. For example,

$$\text{if } a > b,$$
$$\text{then } \frac{a}{-2} < \frac{b}{-2}$$

since $(-\frac{1}{2})(a) < (-\frac{1}{2})(b)$.

The following assertion, which was accepted without proof in Section 2-4, can now be proved using the properties of order.

Theorem. If a is a real number and $a > 0$, then $-a < 0$. Similarly, if $a < 0$, then $-a > 0$.

The proof of the first part of the above theorem is given on the following page.

Prove: If a is a real number and $a > 0$, then $-a < 0$.

PROOF

Statements	*Reasons*
1. a is a real number.	Hypothesis
2. $-a$ is a real number.	Axiom of additive inverses
3. $a > 0$	Hypothesis
4. $a + (-a) > 0 + (-a)$	Addition property of order
5. $a + (-a) = 0$	Axiom of additive inverses
6. $0 + (-a) = -a$	Identity axiom for addition
7. $\therefore 0 > -a$ (or $-a < 0$)	Substitution principle

The second part of this theorem will be proved in Exercise 33 on page 162.

You can use the properties of order to prove additional theorems about the real numbers. For example, you have probably observed that the square of any nonzero real number is a positive real number. You can state this fact as the following theorem.

Theorem. If a is a real number and $a \neq 0$, then $a^2 > 0$.

This theorem can be proved as follows.

Plan: Since $a \neq 0$, the axiom of comparison tells you that there are two cases to consider. Case 1 is $a < 0$, and Case 2 is $a > 0$. The proof of Case 1 is given below. Case 2 will be proved in Exercise 34 on page 162.

PROOF

Statements	*Reasons*
1. a is a real number; $a < 0$	Hypothesis
2. $a \cdot a > 0 \cdot a$	Multiplication property of order
3. $a \cdot a = a^2$	Definition of a^2
4. $0 \cdot a = 0$	Multiplicative property of zero
5. $\therefore a^2 > 0$	Substitution principle

Since $1 \neq 0$ and $1 = 1^2$, the preceding theorem shows that $1 > 0$. Because of this fact and the theorem on page 159, you know that the familiar statement $-1 < 0$ is also true. Indeed, all of the order facts that you have learned in studying arithmetic are consequences of the properties of the real numbers discussed in this section.

Oral Exercises

Name the property that justifies each statement.

1. If $d > 7$, then $d + 2 > 7 + 2$.
2. If $s < -1$, then $s + 8 < -1 + 8$.
3. Of two different real numbers, one must be greater than the other.
4. Any real number that is neither zero nor a positive number must be a negative number.
5. If $5c < 15$, then $\frac{5c}{5} < \frac{15}{5}$.
6. If $z < 4$, then $(-1)z > (-1)4$.
7. If $t > 5$, then $t - 1 > 5 - 1$.
8. If $x < -6$, then $x \div 2 < -6 \div 2$.
9. If j is not less than k and k is not less than j, then $j = k$.
10. Either the statement $a > b$ or the statement $b > a$ is false.
11. If $a < 0$, then $9a < 7a$.
12. If $c > 0$, then $-7c > -12c$.

Written Exercises

Replace each $\underline{?}$ with one of the symbols >, =, or < so that the statement is true for all real values of the variables.

A

1. If $m > n$, then $n \underline{?} m$.
2. If $p < q$, then $q \underline{?} p$.
3. If $a < b$, then $a + 1 \underline{?} b + 1$.
4. If $r > s$, then $r + 5 \underline{?} s + 5$.
5. If $x > y$, then $x - 3 \underline{?} y - 3$.
6. If $m < n$, then $m - 8 \underline{?} n - 8$.
7. If $c < d$, then $2c \underline{?} 2d$.
8. If $x > z$, then $\frac{1}{3}x \underline{?} \frac{1}{3}z$.
9. If $e > f$, then $-4e \underline{?} -4f$.
10. If $b < c$, then $-c \underline{?} -b$.
11. If $s < t$ and $u = 0$, then $su \underline{?} tu$.
12. If $v > w$ and $t = 0$, then $vt \underline{?} wt$.
13. If $k < 0$ and $m < 0$, then $km \underline{?} 0$.
14. If $a > 0$ and $b < 0$, then $ab \underline{?} 0$.
15. If $x > y$ and $w < 0$, then $x + w \underline{?} y + w$.
16. If $a < b$ and $c > 0$, then $a - c \underline{?} b - c$.
17. If $r - 1 < s - 1$, then $r - 5 \underline{?} s - 5$.
18. If $p + 3 > q + 3$, then $p - 6 \underline{?} q - 6$.
19. If $-3g > -18$, then $g \underline{?} 6$.
20. If $-4t < 20$, then $t \underline{?} -5$.

Give the reason that justifies each statement in the given proof.

21. Prove: For all real numbers a, b, c, and d, if $a > b$ and $c > d$, then $a + c > b + d$.

PROOF

1. a, b, c, and d are real numbers; $a > b$
2. $a + c > b + c$
3. $c > d$
4. $b + c > b + d$
5. $\therefore a + c > b + d$

22. Prove: For all positive real numbers a, b, c, and d, if $a > b$ and $c > d$, then $ac > bd$.

PROOF

1. a, b, c, and d are positive real numbers; $a > b$
2. $ac > bc$
3. $c > d$
4. $bc > bd$
5. $\therefore ac > bd$

23. Prove: For all real numbers a and b, if $a < b$, then $a - b < 0$.

PROOF

1. a and b are real numbers; $a < b$
2. $a - b < b - b$
3. $a - b < b + (-b)$
4. $\therefore a - b < 0$

24. Prove: For all real numbers a and b, if $a - b < 0$, then $a < b$.

PROOF

1. a and b are real numbers; $a - b < 0$
2. $a + (-b) < 0$
3. $[a + (-b)] + b < 0 + b$
4. $a + [(-b) + b] < 0 + b$
5. $a + 0 < 0 + b$
6. $\therefore a < b$

If $r > s$, specify the set of all values of t for which each open sentence is a true statement.

B **25.** $rt = st$ **26.** $rt > st$ **27.** $rt^2 > st^2$ **28.** $rt^2 = st^2$ **29.** $\frac{r}{t} < \frac{s}{t}$ **30.** $\frac{r}{t^2} < \frac{s}{t^2}$

31. If $x < y$, specify the set of all values of x and y such that $x^2 < y^2$.

32. If $x < y$, specify the set of all values of x and y such that $x^2 > y^2$.

Write a direct proof of each theorem.

33. If a is a real number and $a < 0$, then $-a > 0$.

34. If a is a real number and $a > 0$, then $a^2 > 0$.

35. If x is a positive real number, then $x^3 > 0$.

36. If x is a negative real number, then $x^3 < 0$.

37. For all real numbers x and y, if $x < 0$ and $y < 0$, then $xy > 0$.

38. For all real numbers x and y, if $x < 0$ and $y > 0$, then $xy < 0$.

39. For all positive real numbers m and n, if $m < n$, then $m^2 < n^2$.

40. For all negative real numbers m and n, if $m < n$, then $m^2 > n^2$.

C 41. For all real numbers x and y, if $x < y$, then $x < \frac{x + y}{2}$.

42. For all real numbers x and y, if $x < y$, then $\frac{x + y}{2} < y$.

Tell whether each statement is true or false for all values of a and b such that $a > b$. If it is false, give an example to justify your answer.

43. $a^2 - b > 0$ 44. $a^2 - b^2 > 0$ 45. $ab - a > 0$ 46. $ab - b^2 > 0$

Computer Exercises For students with computer experience

1. Write a program that will display the order of any two numbers that you input. The output should be in the form of a mathematical sentence that uses one of the symbols $<$, $>$, or $=$.
2. Write a program that will allow you to input any two *different* real numbers and will display the lesser number first, then the greater number. The output should be in the form of an inequality that uses the symbol $<$.
3. Write a program that will allow you to input any *three* different real numbers and will display the numbers in order from least to greatest. The output should be in the form of an inequality that uses the symbol $<$.

4–2 Equivalent Inequalities

Inequalities that have the same solution set over a given domain are called **equivalent inequalities** over that domain. The properties that have been stated in Section 4-1 guarantee that the following transformations of a given inequality always produce an equivalent inequality.

Transformations that Produce an Equivalent Inequality

1. Substituting for either side of the inequality an expression equivalent to that side.
2. Adding to (or subtracting from) each side the same real number.
3. Multiplying (or dividing) each side by the same positive number.
4. Multiplying (or dividing) each side by the same negative number and *reversing* the order of the inequality.

To solve an inequality, you usually try to transform it into a simple equivalent inequality whose solution set can be found by inspection.

EXAMPLE 1 Solve $7(c - 2) + 2 > 2(5c + 9)$ and graph its solution set.

SOLUTION

$$7(c - 2) + 2 > 2(5c + 9)$$

$$7c - 14 + 2 > 10c + 18$$ ← Use the distributive axiom to help simplify each side.

$$7c - 12 > 10c + 18$$

$$7c - 12 + 12 > 10c + 18 + 12$$ ← Add 12 to each side.

$$7c > 10c + 30$$

$$7c - 10c > 10c + 30 - 10c$$ ← Subtract $10c$ from each side.

$$-3c > 30$$

$$\frac{-3c}{-3} < \frac{30}{-3}$$ ← Divide each side by -3 and reverse the direction of the inequality.

$$c < -10$$

CONDENSED SOLUTION

$$7(c - 2) + 2 > 2(5c + 9)$$

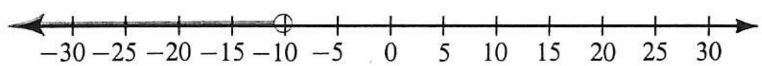
$$7c - 12 > 10c + 18$$

$$7c > 10c + 30$$

$$-3c > 30$$

$$c < -10$$

$\therefore$ the solution set is $\{c\colon c < -10\}$.



One method used to check that an equation has been solved correctly is to test each member of the solution set to see if it satisfies the original equation. This method of checking can be used for equations and inequalities if the solution set is finite, but it cannot be used if the solution set is infinite.

The solution set in the previous example, $\{c\colon c < -10\}$, is an infinite set. You cannot test every number in the solution set, but errors can usually be found by testing one value in each region of the graph.

Try $c = -11$. Since -11 is in the solution set, the original inequality should be true when $c = -11$.

$$7(c - 2) + 2 > 2(5c + 9)$$

$$7(-11 - 2) + 2 > 2[5(-11) + 9]$$

$$-89 > -92 \qquad \text{True}$$

Try $c = -9$. Since -9 is not in the solution set, the original inequality should be false when $c = -9$.

$$7(c - 2) + 2 > 2(5c + 9)$$

$$7(-9 - 2) + 2 > 2[5(-9) + 9]$$

$$-75 > -72 \qquad \text{False}$$

To solve an inequality, you take the same steps used to solve equations.

1. Simplify each side of the inequality.
2. Use the inverse operations to undo any indicated additions or subtractions.
3. Use the inverse operations to undo any indicated multiplications or divisions.

Certain inequalities are true for all real numbers, and others have no solution.

EXAMPLE 2 Solve. **a.** $2x > 2(x + 1)$ **b.** $3y < 3(y + 2)$

SOLUTION **a.** $2x > 2(x + 1)$

$2x > 2x + 2$

$0 > 2$

Since the given inequality is equivalent to the false statement $0 > 2$, the inequality has no solution.

$\therefore$ the solution set is $\emptyset$.

b. $3y < 3(y + 2)$

$3y < 3y + 6$

$0 < 6$

Since the given inequality is equivalent to the true statement $0 < 6$, the inequality is satisfied by every real number.

$\therefore$ the solution set is $\mathcal{R}$.

Oral Exercises

State the transformation used to transform the first inequality into the second.

1. $b + 5 < 8$; $b < 3$
2. $z - 6 > 5$; $z > 11$
3. $w - 7 > -13$; $w > -6$
4. $p + 12 < -7$; $p < -19$
5. $8t > 32$; $t > 4$
6. $3w < -18$; $w < -6$
7. $\frac{e}{3} < -4$; $e < -12$
8. $\frac{h}{4} > 2$; $h > 8$
9. $-5t > 30$; $t < -6$
10. $-3d < 12$; $d > -4$
11. $-8x < -4$; $x > \frac{1}{2}$
12. $-7a > -14$; $a < 2$

Written Exercises

Solve each inequality and graph its solution set.

A

1. $a - 13 > -14$
2. $n + 11 < 17$
3. $2 < c + 8$
4. $-15 > t - 9$
5. $11b < -121$
6. $15p > 105$
7. $4 > \frac{m}{3}$
8. $6 < \frac{n}{2}$
9. $\frac{2}{3}t > -4$
10. $\frac{3}{5}x < 0$
11. $0 > -3w$
12. $-5p > -20$
13. $29 < 5c - 6$
14. $132 > 7q + 6$
15. $3 - d > 16$
16. $0 > -4 - r$
17. $23 < 5 - 3e$
18. $5 - 6s > 71$
19. $-2 < 2 + \frac{f}{3}$
20. $\frac{t}{11} + 6 > 6$
21. $\frac{4}{5}u - 3 > 9$
22. $16 < 10 - \frac{2}{3}g$
23. $8 > \frac{h - 3}{2}$
24. $\frac{v + 1}{-3} > -1$

Solve.

B

25. $5i - 9 < 3 - i$
26. $11 - 2w < 3w - 9$
27. $3j + 25 > 6j - 23 + j$
28. $12x - 11 - 10x < 9 + 7x + 10$
29. $3k - 7(k + 5) - 5 < 0$
30. $11 + 6(y - 2) - 5y < 0$
31. $-4(t - 4) < 5(t + 3)$
32. $8(z + 5) > -(20 - 3z)$
33. $7(a + 4) - 13 < 13(3 + a) + 12$
34. $7 - 2(m - 4) < 5(1 - 2m)$
35. $3(1 + 3t) - 7 > 4(3t + 2) - 3t$
36. $3(d - 8) < 5(d - 1) + 2(10 - d)$

C

37. $5[3(s - 6) - 2(3s - 5)] + 3[3(s + 5) + 8s + 7] < 2(4s + 3)$
38. $5[3(2 - 3f) - 2(5 - f)] - 6[5(f - 2) - 2(4f - 3)] < 3f + 19$
39. $\frac{2}{3}\left[2(b - 9) - \frac{1}{2}(b - 4)\right] < -\frac{5}{6}\left[3(b + 2) - 2(b + 1)\right]$
40. $\frac{1}{2}\left[\frac{1}{5}(a - 2) - \frac{2}{5}(a - 3)\right] > \frac{3}{4}\left[2a - \frac{1}{3}(4 + 5a)\right]$

Computer Exercises For students with computer experience

Write a program that will solve an inequality of the form $ax + b < c$, $a \neq 0$, when you input values for a, b, and c. Be careful to account for negative values of a. RUN the program to solve each of the following.

1. $4x + 3 < 15$
2. $3y - 2 < 16$
3. $-2z + 5 < 17$
4. $-5w - 2 < 18$

Modify the program that you wrote for Exercises 1–4 to solve an inequality of either form $ax + b < c$ or $ax + b > c$, $a \neq 0$. RUN the program to solve each of the following.

5. $4a + 5 < 21$
6. $4a + 5 > 21$
7. $-2b + 9 < 15$
8. $-5c - 7 > 3$

Modify the program that you wrote for Exercises 5–8 to solve any inequality of the form $ax + b < cx + d$ or $ax + b > cx + d$. Be careful to account for inequalities whose solution set is empty and for inequalities whose solution set is $\mathcal{R}$. RUN the program to solve each of the following.

9. $5m + 1 < 3m + 11$ **10.** $2n - 4 > 7n - 9$ **11.** $3 + 4r > 9 + 2r$

12. $8 + s < 4 - s$ **13.** $-4p + 7 < 6 - 4p$ **14.** $2q - 9 > 2(q - 5)$

4–3 Intersection and Union of Sets

Just as you can perform operations on real numbers, you can also perform operations on sets. *Intersection* and *union* are two examples of set operations.

The **intersection** of any two sets S and T is the set consisting of the members belonging to *both* S and T. For example, if

$$S = \{0, 2, 4, 6\} \quad \text{and} \quad T = \{0, 4, 8, 12\},$$

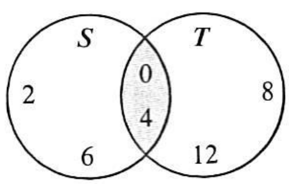
the intersection of S and T is the set $\{0, 4\}$. To indicate the intersection of these sets, you use the symbol $\cap$ and write

$$S \cap T = \{0, 4\}.$$

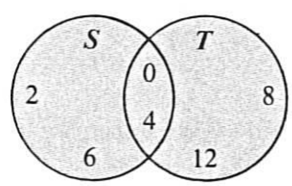
The **union** of any two sets S and T is the set consisting of the members belonging to *at least one* of the sets S and T. The union of the sets S and T just given is the set $\{0, 2, 4, 6, 8, 12\}$. To indicate the union of these sets, you use the symbol $\cup$ and write

$$S \cup T = \{0, 2, 4, 6, 8, 12\}.$$

Recall from Section 1-2 that you can use Venn diagrams to show how sets are related. In the Venn diagrams that follow, shading is used as shown to represent the intersection and union of the sets S and T.



$S \cap T = \{0, 4\}$



$S \cup T = \{0, 2, 4, 6, 8, 12\}$

Sets that have no members in common are called **disjoint sets.** For example, if

$$R = \{1, 3, 5, 7, 9\} \quad \text{and} \quad S = \{0, 2, 4, 6\},$$

R and S are disjoint sets. Note that the intersection of two disjoint sets is the empty set. Thus you can write

$$R \cap S = \emptyset.$$

When there is more than one set operation in an expression, parentheses are used to indicate which operations are to be performed first.

EXAMPLE 1 Let $A = \{-2, -1, 1, 2\}$, $B = \{0, 1, 3\}$, and $C = \{1, 2, 3, 4\}$. Specify by roster the set $A \cup (B \cap C)$.

SOLUTION First specify $B \cap C$.

$$B \cap C = \{0, 1, 3\} \cap \{1, 2, 3, 4\} = \{1, 3\}$$

Then specify $A \cup (B \cap C)$.

$$A \cup (B \cap C) = \{-2, -1, 1, 2\} \cup \{1, 3\} = \{-2, -1, 1, 2, 3\}$$

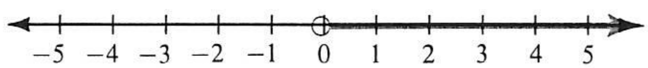
The operations of intersection and union can be performed on infinite sets as well as on finite sets. In the next example, a number line is used to represent the intersection and the union of infinite sets of numbers.

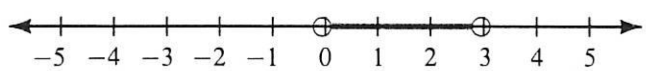
EXAMPLE 2 Let R = {the real numbers between −3 and 3} and S = {the positive real numbers}. Graph each of the following sets.

a. R **b.** S **c.** $R \cap S$ **d.** $R \cup S$

SOLUTION

a. 

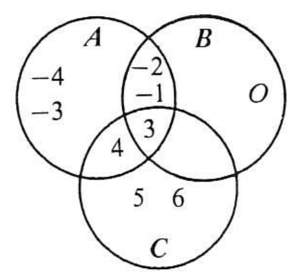
b. 

c. 

d. 

Oral Exercises

Exercises 1–12 refer to the Venn diagram at the right. Specify each set by roster.



Exs. 1–12

1. $A \cap B$
2. $B \cap C$
3. $A \cup B$
4. $B \cup C$
5. $C \cap A$
6. $C \cup A$
7. $(A \cap B) \cap C$
8. $A \cup (B \cup C)$
9. $(A \cap B) \cup C$
10. $A \cap (B \cup C)$
11. $(A \cap C) \cap (B \cap C)$
12. $(A \cup B) \cap (B \cap C)$

Written Exercises

Specify the intersection and the union of the given sets.

A

1. $\{-3, -1, 1, 3\}, \{-1, 0, 1\}$
2. $\{-10, -6, -2\}, \{-2, 2, 6\}$
3. $\{-6, -4, 1\}, \{2, 4, 6\}$
4. $\{2, 3, 4\}, \{2, 3, 4\}$
5. $\{-2, 1, 8, 10\}, \{-2, 1, 8, 10\}$
6. $\{-1, -3, 5\}, \{1, 3, -5\}$
7. $\{1, 2, 3, 4\}, \emptyset$
8. $\{0\}, \emptyset$
9. {the even whole numbers}, $\{1, 2, 3, 4\}$
10. {the even integers}, {the odd integers}
11. {the integers less than 4}, {the integers greater than 6}
12. {the integers greater than -2}, {the integers greater than -5}
13. {the natural numbers less than 5}, {the natural numbers between 1 and 6}
14. {the whole numbers less than or equal to 3}, {5 and the natural numbers less than 5}
15. {the whole numbers less than 8}, {the whole numbers less than or equal to 6}
16. {the negative integers greater than -5}, {the integers between -2 and 2}

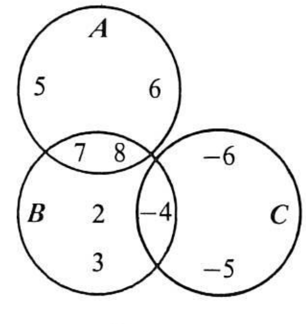
Graph each of the following when *R* and *S* are the given sets.

a. $R \cup S$ **b.** $R \cap S$

B

17. R = {the real numbers between -2 and 6}, S = {the positive real numbers}
18. R = { the real numbers between -5 and 5}, S = {the negative real numbers}
19. R = {the real numbers greater than $-\frac{1}{2}$}, S = {the real numbers less than $2\frac{1}{2}$}
20. R = {the real numbers greater than or equal to 4.5}, S = {the real numbers less than or equal to 5.5}
21. R = {the real numbers greater than or equal to 4}, S = {the real numbers less than -2}
22. R = {the real numbers greater than -5}, S = {the real numbers less than -5}
23. R = {the real numbers greater than -3}, S = {the real numbers greater than -6}
24. R = {the real numbers less than 3}, S = {the real numbers less than -1}

In each exercise, copy the Venn diagram shown at the right. Then shade the region that represents the given set.



Exs. 25–34

25. $A \cap (B \cup C)$

26. $(A \cap B) \cup C$

27. $(A \cup C) \cap B$

28. $A \cup (C \cap B)$

29. $A \cup (B \cap C)$

30. $C \cap (A \cup B)$

31. $(A \cup B) \cap (A \cup C)$

32. $(C \cap A) \cup (C \cap B)$

33. $(A \cap B) \cup (A \cap C)$

34. $(A \cup B) \cap (A \cap C)$

Replace each _?_ with one of the symbols R, S, T, or $\emptyset$ to make a true statement.

35. If $R \subset S$, then $R \cap S =$ _?_.

36. If $T \subset S$, then $S \cup T =$ _?_.

37. If R and S are disjoint sets and $T \subset S$, then $R \cap T =$ _?_.

38. If $R \subset S$ and $S \subset T$, then $(R \cup S) \cup T =$ _?_.

Let A, B, and C be subsets of $U = \{0, 1, 2, 3, 4, 5, 6\}$. Given the following information about sets A, B, and C, specify B by roster.

C **39.** $A = \{1, 2\}$, $A \cap B = \{1\}$, $A \cup B = \{1, 2, 3, 6\}$

40. $A = \{1, 2, 3, 6\}$, $A \cap B = \{1, 6\}$, $A \cup B = \{1, 2, 3, 5, 6\}$

41. $A \cap A = \emptyset$, $C \cup A = \{6\}$, $B \cup C = U$, $B \cap C = \emptyset$

42. $A = \{3, 5\}$, $B \cap C = \{3\}$, $B \cup C = \{1, 2, 3, 4, 6\}$, $A \cup C = \{1, 3, 4, 5\}$

43. $A \cap C = \{0, 4\}$, $A \cap B = \{0, 3\}$, $B \cap C = \{0, 6\}$, $A \cup C = \{0, 1, 2, 3, 4, 6\}$, $B \cup C = \{0, 2, 3, 4, 5, 6\}$

44. $A \cup B = \{1, 2, 3, 4, 5\}$, $A \cup C = \{1, 3, 4, 5\}$, $A \cap C = A$, $A \cap B = \emptyset$, $B \cap C = \{3, 5\}$

4–4 Combined Inequalities

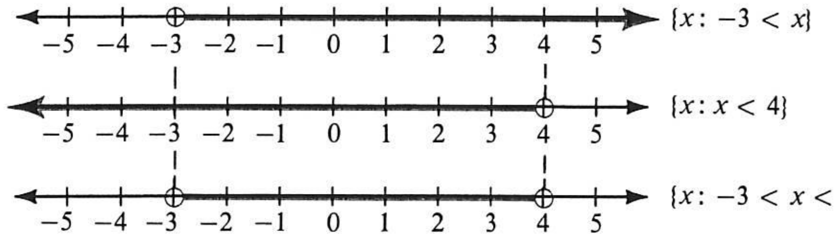
A sentence formed by joining two sentences with the word *and* is called a **conjunction.** An example of a conjunction is

$$-3 < x \quad \text{and} \quad x < 4.$$

For a conjunction to be true, *both* of the joined sentences must be true. Therefore, the solution set of the conjunction $-3 < x$ and $x < 4$ is the *intersection* of the solution set of $-3 < x$ and the solution set of $x < 4$. That is,

$$\{x: -3 < x\} \cap \{x: x < 4\} = \{x: -3 < x < 4\},$$

as shown by the diagram on the following page.



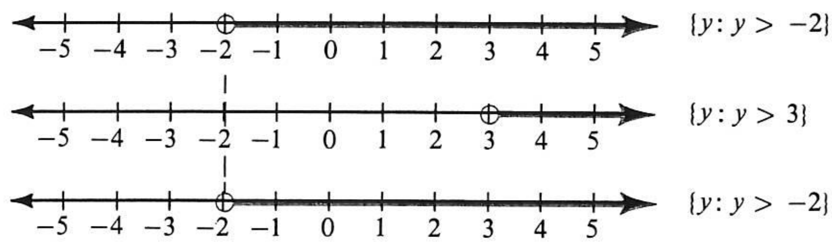
A sentence formed by joining two sentences with the word *or* is called a **disjunction.** An example of a disjunction is

$$y > -2 \qquad \text{or} \qquad y > 3.$$

For a disjunction to be true, *at least one* of the joined sentences must be true. Therefore, the solution set of the disjunction $y > -2$ or $y > 3$ is the *union* of the solution set of $y > -2$ and the solution set of $y > 3$. That is,

$$\{y: y > -2\} \cup \{y: y > 3\} = \{y: y > -2\},$$

as shown by the following diagram.



EXAMPLE 1 Solve.

a. $x < -2$ and $x \geq 5$ **b.** $y < 3$ or $y \geq -1$

SOLUTION **a.** Since the combined inequality is a *conjunction*, the solution set consists of all those numbers that satisfy *both* the joined sentences. However, there is no real number that satisfies both $x < -2$ and $x \geq 5$. $\therefore$ the solution set is $\emptyset$.

b. Since the combined inequality is a *disjunction*, the solution set consists of all those numbers that satisfy *at least one* of the joined sentences. However, every real number satisfies either $y < 3$ or $y \geq -1$. $\therefore$ the solution set is $\mathcal{R}$.

Can you describe the solution set of the open sentence

$$-2 \leq x + 3 < 7?$$

This open sentence is equivalent to the conjunction

$$-2 \leq x + 3 \qquad \text{and} \qquad x + 3 < 7.$$

Therefore, to solve the given open sentence you must find the values of x that satisfy both of the inequalities $-2 \leq x + 3$ and $x + 3 < 7$. Example 2 shows two methods of solving this open sentence.

EXAMPLE 2 Solve $-2 \le x + 3 < 7$ and graph its solution set.

SOLUTION 1

$-2 \le x + 3$	and	$x + 3 < 7$
$-2 - 3 \le x + 3 - 3$	|	$x + 3 - 3 < 7 - 3$
$-5 \le x$	and	$x < 4$

$\therefore$ the solution set is $\{x\colon -5 \le x < 4\}$.

SOLUTION 2 It is possible to solve $-2 \le x + 3 < 7$ more compactly as follows.

$$-2 \le x + 3 < 7$$
$$-2 - 3 \le x + 3 - 3 < 7 - 3$$
$$-5 \le x < 4$$

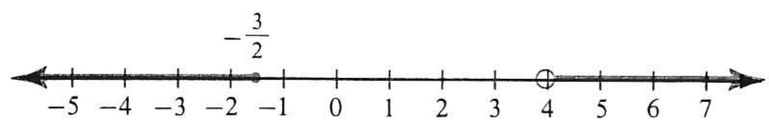
Again, the solution set is $\{x\colon -5 \le x < 4\}$.

EXAMPLE 3 Solve the open sentence $2z - 3 \le -6$ or $3z - 12 > 0$ and graph its solution set.

SOLUTION

$2z - 3 \le -6$	or	$3z - 12 > 0$
$2z \le -3$	|	$3z > 12$
$z \le -\frac{3}{2}$	or	$z > 4$

$\therefore$ the solution set is $\left\{z\colon z \le -\frac{3}{2} \text{ or } z > 4\right\}$.



Note that the familiar open sentence $y \ge 3$ is an example of a disjunction since it means $y > 3$ or $y = 3$. Similarly, $y \le 5$ is a disjunction since it means $y < 5$ or $y = 5$.

Oral Exercises

Replace each __?__ with the word or phrase that makes a true statement.

1. In a conjunction, sentences are joined by the word __?__.
2. In a disjunction, sentences are joined by the word __?__.
3. A conjunction of two sentences is true provided __?__ of the joined sentences (is/are) true.
4. A disjunction of two sentences is true provided __?__ of the joined sentences (is/are) true.

Tell whether each statement is true or false.

5. $5 < 9$ or $0 > -1$

6. $4 < 10$ and $6 < 11$

7. $-5 > -4$ and $3 < 2$

8. $5 > 6$ or $-2 = 2$

9. $-2 \leq -1$ or $0 \leq -1$

10. $0 < -5$ and $6 > 5$

11. $5.6 > 0.3$ and $0.33 < 0.3$

12. $3.4 < 0$ or $1.6 > 0$

Match each open sentence with the graph of its solution set.

13. $x \geq -1$ or $x < 3$

14. $x \geq -1$ and $x < 3$

15. $x \leq -1$ or $x \geq 3$

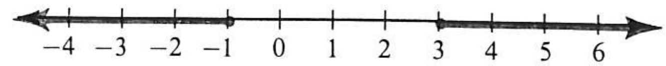
16. $x \leq -1$ and $x \geq 3$

17. $x < 3$ or $x \leq -1$

18. $x < 3$ and $x \leq -1$

19. $x < 3$ or $x > 3$

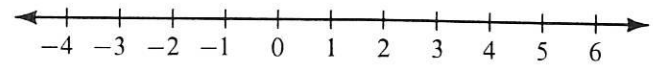
20. $x \leq -1$ and $x \geq -1$

a. 

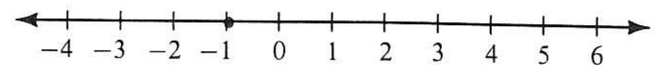
b. 

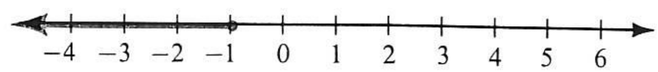
c. 

d. 

e. 

f. 

g. 

h. 

Written Exercises

Solve each open sentence and graph each solution set that is not empty.

A

1. $a < -3$ or $a > 1$

2. $m < 5$ and $m > -3$

3. $e \geq 2$ and $e < -1$

4. $n > -2$ or $n < 6$

5. $t > -4$ or $t < 1$

6. $r > 4$ and $r < 0$

7. $z > -1$ and $z < 2$

8. $t < 3$ or $t > 5$

9. $w > -3$ and $w > 0$

10. $p > -2$ or $p > 1$

11. $-7 \leq x + 5 < 2$

12. $3 < c - 3 < 5$

13. $z + 1 > 2$ or $z + 1 < -6$

14. $5x < -20$ and $3x > -18$

15. $2a > -6$ and $3a \leq 15$

16. $20 \geq 4c \geq -2$

17. $-10 \leq 2t \leq 12$

18. $3a + 1 > -8$ and $a - 4 \leq 2$

19. $2x - 1 < 5$ and $3x + 2 > -4$

20. $-21 \leq 4n - 5 < -1$

21. $15 \geq 2d - 1 > -9$

22. $3z - 5 < -1$ or $3z - 5 > -4$

23. $-6 < \frac{1}{2}r - 4 < -3$

24. $-3 \leq \frac{2}{3}t - 5 < -1$

Solve.

B 25. $-2 < -3k \leq 3$

26. $-4 \leq -2t \leq -1$

27. $5 - 2h < 1$ and $5 - 2h > -7$

28. $5 - r < -6$ and $5 - r \geq -2$

29. $3 - 2y > -2$ or $3y + 6 < 8$

30. $6 - 3x \leq 2$ or $2 - x > 4$

31. $-8 < 2 - 3g < 10$

32. $5 \geq -2t + 1 \geq -3$

33. $6 \geq 3 - \frac{1}{2}x \geq -2$

34. $7 \leq 1 - \frac{3}{5}z \leq 8$

35. $7d > 2d + 10$ and $2d > 5d - 15$

36. $4t - 3 > t - 9$ and $6 - 5t > 1 - 3t$

37. $4j + 3 > 13 - j$ or $16 - 3j \geq -8$

38. $26 - 3v > v - 14$ or $1 - 8v < 31 - 5v$

C 39. $2e - 3 < 3e + 1 < 2e + 2$

40. $8 - 4v \geq 7 - 5v \geq 5 - 4v$

41. $6 - 3x \geq 4 - 2x > 5 - 4x$

42. $3t - 5 \leq 7t - 1 \leq 6t + 5$

43. $2d - 4 < 2d + 5$ and $2d + 1 > 2d - 4$

44. $3p - 5 > 3p + 6$ or $3p - 5 < 3p - 6$

Let $A = \{x: x \geq -2\}$, $B = \{x: x < 5\}$, and $C = \{x: -3 \leq x < 3\}$. Specify each of the following sets.

45. $A \cap (B \cap C)$

46. $A \cup (B \cup C)$

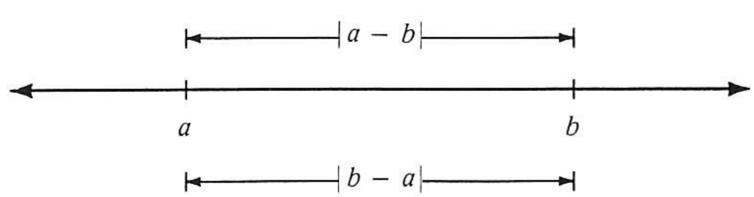
47. $(A \cup B) \cap (A \cup C)$

48. $(B \cap C) \cup (B \cap A)$

4–5 Absolute Values in Open Sentences

In Section 2-4, you saw that the expression $|x|$ could be interpreted as the distance between the graph of x and the origin on a number line. Similarly, the expression $|a - b|$ can be interpreted as the distance between the graphs of a and b. Since the distance from the graph of a to the graph of b is the same as the distance from the graph of b to the graph of a, you can write

$$|a - b| = |b - a|.$$



Note that, for simplicity, you usually speak of the distance between a

and b instead of the distance between the graphs of a and b.

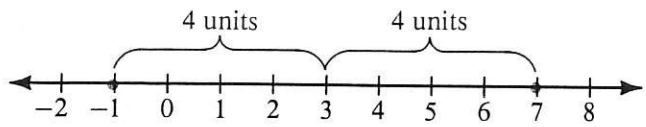
The following example shows how the relationship between absolute value and distance can be used in solving open sentences involving absolute value.

EXAMPLE 1 Solve.

a. $|x - 3| = 4$ **b.** $|x - 3| > 4$ **c.** $|x - 3| < 4$ **d.** $|x + 3| \geq 4$

SOLUTION **a.** $|x - 3| = 4$

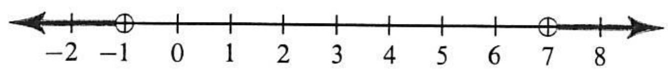
The distance between x and 3 is 4 units.



$\therefore$ the solution set is $\{-1, 7\}$.

b. $|x - 3| > 4$

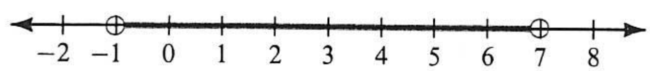
The distance between x and 3 is more than 4 units.



$\therefore$ the solution set is $\{x\colon x < -1 \text{ or } x > 7\}$

c. $|x - 3| < 4$

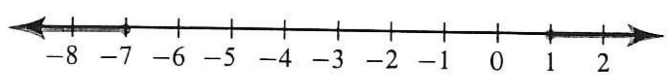
The distance between x and 3 is less than 4 units.



$\therefore$ the solution set is $\{x\colon -1 < x < 7\}$.

d. $|x + 3| \geq 4$ is equivalent to $|x - (-3)| \geq 4$.

The distance between x and -3 is 4 units or more than 4 units.



$\therefore$ the solution set is $\{x\colon x \geq 1 \text{ or } x \leq -7\}$.

Some open sentences involving absolute value can be expressed as conjunctions; others can be expressed as disjunctions. For example:

$|2x + 7| = 9$ is equivalent to $2x + 7 = 9$ or $2x + 7 = -9$.
$|2x + 7| > 9$ is equivalent to $2x + 7 > 9$ or $2x + 7 < -9$.
$|2x + 7| < 9$ is equivalent to $-9 < 2x + 7 < 9$.

Often you use an equivalent conjunction or disjunction in solving an open sentence involving absolute value.

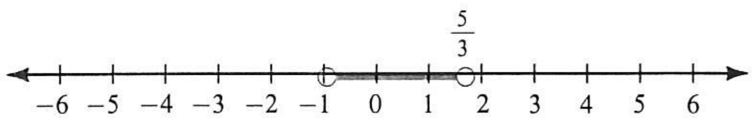
EXAMPLE 2 Solve $|1 - 3y| < 4$ and graph the solution set.

SOLUTION $|1 - 3y| < 4$ is equivalent to $-4 < 1 - 3y < 4$.

$$-4 < 1 - 3y < 4$$
$$-4 - 1 < 1 - 3y - 1 < 4 - 1$$
$$-5 < -3y < 3$$
$$\frac{-5}{-3} > \frac{-3y}{-3} > \frac{3}{-3}$$
$$\frac{5}{3} > y > -1$$

$\therefore$ the solution set is $\left\{y: \frac{5}{3} > y > -1\right\}$.

Another way of expressing this solution is $\left\{y: -1 < y < \frac{5}{3}\right\}$.



Since $|1 - 3y| = |3y - 1|$, the inequality in Example 2 can also be solved by finding the solution set of the equivalent inequality

$$|3y - 1| < 4.$$

You may find it easier to solve $|3y - 1| < 4$ than the original inequality.

EXAMPLE 3 Solve $2 + 3|2z - 1| > 8$ and graph the solution set.

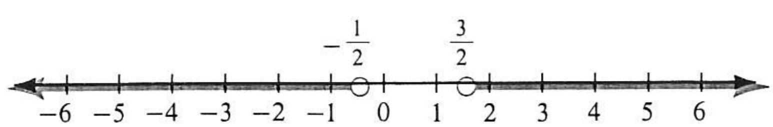
SOLUTION First transform the inequality into an equivalent inequality with the absolute value term alone as one side.

$$2 + 3|2z - 1| > 8$$
$$3|2z - 1| > 6$$
$$|2z - 1| > 2$$

Then solve the equivalent inequality.

$$2z - 1 > 2 \quad \text{or} \quad 2z - 1 < -2$$
$$2z > 3 \qquad\qquad 2z < -1$$
$$z > \frac{3}{2} \quad \text{or} \quad z < -\frac{1}{2}$$

$\therefore$ the solution set is $\left\{z: z > \frac{3}{2} \text{ or } z < -\frac{1}{2}\right\}$.



Oral Exercises

Replace each __?__ with one of the words *and* or *or* to make a true statement.

1. $|d| = 4$ is equivalent to $d = -4$ __?__ $d = 4$.
2. $|m| > 5$ is equivalent to $m < -5$ __?__ $m > 5$.
3. $|w| \leq 3$ is equivalent to $-3 \leq w$ __?__ $w \leq 3$.
4. $|c| - 2 < 1$ is equivalent to $|c| < 3$, which is equivalent to $-3 < c$ __?__ $c < 3$.

Match each open sentence with its solution set.

5. $|x + 4| \leq 6$
6. $|x - 4| \leq 6$
7. $|4 - x| > 6$
8. $|4 - x| < 6$
9. $|x + 4| \geq -6$
10. $|x + 4| \leq -6$

a. $\emptyset$
b. $\mathcal{R}$
c. $\{x: -2 < x < 10\}$
d. $\{x: -10 \leq x \leq 2\}$
e. $\{x: -2 \leq x \leq 10\}$
f. $\{x: x > 10 \text{ or } x < -2\}$

State a conjunction or a disjunction equivalent to each open sentence.

11. $|x - 3| = 5$
12. $|m + 1| = 8$
13. $|y| > 3$
14. $|n| \leq 2$
15. $|u - 4| < 2$
16. $|r - 1| \geq 3$

Written Exercises

Write an equation or inequality involving absolute value to describe the set of all real numbers *x* that are at the given location on a number line.

A

1. 2 units from 5
2. 3 units from -2
3. less than 3 units from -1
4. more than 4 units from 7
5. no more than 6 units from 1
6. no less than 2 units from -4
7. at least 5 units from 10
8. 6 units from a
9. more than b units from 5
10. less than or equal to a units from b

Solve each open sentence and graph each solution set that is not empty.

11. $|t - 4| = 6$
12. $|x - 5| = 3$
13. $|a + 1| = 2$
14. $|w + 5| = 6$
15. $|v - 6| \leq 6$
16. $|p - 2| \geq 4$
17. $|c - 8| > -2$
18. $|t - 4| < -4$
19. $|10 - v| \leq 2$
20. $|1 - r| \geq 3$
21. $|1 - x| > 7$
22. $|-p + 4| < 10$
23. $|5v| \geq 15$
24. $\left|\frac{1}{2}t\right| \leq 1$
25. $\left|-\frac{1}{3}w\right| \leq 2$
26. $|-6x| > -3$
27. $|2w - 7| \geq 3$
28. $|3q - 1| < 5$
29. $|6 - 3c| \geq 2.1$
30. $|4 - 2p| \leq -6.4$
31. $|5 - 2a| \geq -5$
32. $|1 - 3x| < 10$
33. $\left|\frac{1}{2}x + 3\right| < \frac{5}{2}$
34. $\left|\frac{2}{3}t - 4\right| > \frac{1}{6}$

Solve.

B 35. $5 + |3 - c| \geq 8$

36. $3 + |c - 1| \leq 12$

37. $13 + |4 - z| \leq 9$

38. $12 + |2 - k| > 8$

39. $6 - |2c + 3| > -4$

40. $5 - |4k - 1| \leq -7$

41. $3|x - 6| - 4 \leq 11$

42. $2|3a - 5| + 1 > 7$

43. $|8 - (3 - d)| - 5 > -1$

44. $|5 - (v - 3)| + 8 < 15$

Write an equation or inequality involving absolute value whose solution set is the set given.

45. $\{-2, 2\}$

46. $\{-6, 6\}$

47. $\emptyset$

48. $\mathcal{R}$

49. $\{5, 9\}$

50. $\{6, 12\}$

51. $\{-8, -12\}$

52. $\{-8, 2\}$

53. $\{x: -4 < x < 4\}$

54. $\{x: x > 2 \text{ or } x < -2\}$

55. $\{x: x > 5 \text{ or } x < -1\}$

56. $\{x: -3 < x < 11\}$

Specify all the real numbers *x* for which each statement is true.

C 57. a. $|x + 3| < x + 3$ b. $|x + 3| = x + 3$ c. $|x + 3| > x + 3$

58. a. $|x - 2| < x - 2$ b. $|x - 2| = x - 2$ c. $|x - 2| > x - 2$

59. a. $|x^2| = x^2$ b. $|-x^2| = x^2$ c. $|-x^3| = -x^3$

60. a. $|-3x| = -3x$ b. $|-3x| = -3|x|$ c. $-|3x| = -3x$

61. a. $|-2x + 3| < |-2x| + 3$
    b. $|-2x + 3| = |-2x| + 3$
    c. $|-2x + 3| > |-2x| + 3$

62. a. $|-2x - 3| < |-2x| - 3$
    b. $|-2x - 3| = |-2x| - 3$
    c. $|-2x - 3| > |-2x| - 3$

Solve. *Hint*: Using a number line may be helpful.

63. $3 < |d| < 5$

64. $1 < |c| < 6$

65. $-5 < |a| < -1$

66. $-4 < |t| < 4$

Self-Test 1

VOCABULARY

axiom of comparison (p. 157)
transitive property of order (p. 158)
addition property of order (p. 158)
multiplication property of order (p. 159)
equivalent inequalities (p. 163)
intersection (p. 167)
union (p. 167)
disjoint sets (p. 167)
conjunction (p. 170)
disjunction (p. 171)

Give the reason that justifies each statement, if c is a real number.

1. If $3 - 2c < 17$, then $(3 - 2c) - 3 < 17 - 3$. *Obj. 1, p. 157*
2. If $-2c < 14$, then $c > -7$.

Solve.

3. $2x + 3 > -5$ 	4. $5 - 4y \leq 9$ 	*Obj. 2, p. 157*

Let A = {the integers between −5 and 5} and B = {the whole numbers less than or equal to 6}. Specify each set.

5. $A \cap B$ 	6. $A \cup B$ 	*Obj. 3, p. 157*

Solve each open sentence and graph its solution set.

7. $-7 \leq t + 3 < 5$ 	8. $m - 2 > 3$ or $m + 5 < -6$ 	*Obj. 4, p. 157*

9. $|z - 1| = 6$ 	10. $|r + 4| = 2$ 	*Obj. 5, p. 157*

11. $|p - 5| > 3$ 	12. $|2x - 3| \leq 6$

Check your answers with those at the back of the book.

Inequalities and Equations in Solving Problems

OBJECTIVES for Sections 4-6 through 4-10:
1. *To solve word problems involving consecutive integers, consecutive multiples, angle relationships, uniform motion, and mixtures.*
2. *To solve word problems involving inequalities.*

4–6 Problems about Integers

If you count by ones from any given integer, you obtain **consecutive integers**. The following are examples of consecutive integers.

five consecutive integers	3, 4, 5, 6, 7
four consecutive integers	−2, −1, 0, 1
three consecutive integers when x is the least integer	$x, x + 1, x + 2$
three consecutive integers when x is the middle integer	$x - 1, x, x + 1$
three consecutive integers when x is the greatest integer	$x - 2, x - 1, x$

Sometimes an inequality is needed to translate a word problem into the language of algebra. Consider, for instance, the following problem about consecutive integers.

EXAMPLE Find all sets of four consecutive positive integers such that the greatest integer in the set is more than twice the least integer in the set.

SOLUTION

Step 1 The problem asks for sets of four consecutive integers that are positive. The greatest integer in the set is more than twice the least. All such sets are to be found.

Step 2 Let x = the least of the four positive integers. Then:
$x + 1$ = the next greater integer,
$x + 2$ = the next integer,
$x + 3$ = the greatest integer

Step 3 The greatest integer is more than twice the least integer.

$$\underbrace{\text{The greatest integer}}_{x+3}\ \underbrace{\text{is more than}}_{>}\ \underbrace{\text{twice the least integer.}}_{2x}$$

Step 4
$$x + 3 > 2x$$
$$3 > x$$

The problem states that the integers must be positive. Since $x < 3$, the only possible choices for x are 1 and 2.
Thus, the only possible sets of four consecutive positive integers are $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 4, 5\}$.

Step 5 In each set, is the greatest integer more than twice the least?

In A: $4 \overset{?}{>} 2(1)$ In B: $5 \overset{?}{>} 2(2)$
$4 > 2$ ✓ $5 > 4$ ✓

Are these two sets *all* the solutions?
Check the next possible set of four consecutive integers, $\{3, 4, 5, 6\}$, to see if the condition is satisfied.

$6 \overset{?}{>} 2(3)$
$6 > 6$ False

No other set will satisfy the condition.
$\therefore$ the required sets are $\{1, 2, 3, 4\}$ and $\{2, 3, 4, 5\}$.

The product of any real number and an integer is called a **multiple** of that real number. If you multiply a given number n by consecutive integers, you obtain **consecutive multiples** of n. For example, if x is an integer, then

$$\ldots, 8(x - 2), 8(x - 1), 8x, 8(x + 1), 8(x + 2), \ldots$$

are consecutive multiples of 8.

The multiples of 2 are the *even integers*:

$$\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots$$

The integers that are not even are the *odd integers*:

$$\ldots, -5, -3, -1, 1, 3, 5, \ldots$$

If you count by twos from any given even integer, you obtain **consecutive even integers**. If you count by twos from any given odd integer, you obtain **consecutive odd integers**. In general,

$$\ldots, a - 4, a - 2, a, a + 2, a + 4, \ldots$$

will represent *consecutive even integers* if a is an even integer and will represent *consecutive odd integers* if a is an odd integer.

Oral Exercises

Let c = 8. Express each of the given numbers in terms of c.

1. 8, 9, 10
2. 7, 8, 9
3. 10, 12, 14
4. 2, 4, 6
5. 3, 5, 7
6. 7, 9, 11

Let e = −5. Express each of the given numbers in terms of e.

7. −5, −4, −3
8. −6, −5, −4
9. −5, −3, −1
10. −7, −5, −3
11. −10, −9, −8
12. −6, −4, −2
13. Let z represent an even integer.
    a. What are the next two greater even integers?
    b. What is the preceding even integer?
14. Let $w - 3$ represent an integer.
    a. What are the next two greater integers?
    b. What are the two preceding integers?
15. Let $7r$ represent a multiple of 7.
    a. What are the next three greater multiples of 7?
    b. What is the preceding multiple of 7?
16. Let k represent an odd integer.
    a. What is the next greater odd integer?
    b. What is the next greater integer?
    c. What is the preceding odd integer?
    d. What is the preceding integer?
17. Let n represent an integer.
    a. Tell whether $2n$ is an even integer or an odd integer.
    b. Tell whether $2n + 1$ is an even integer or an odd integer.
18. Let d represent an integer. Can you tell whether $d + 1$ is an even integer or an odd integer? Explain.

Translate each word sentence into a mathematical sentence.

19. The sum of two consecutive integers is 71.
20. The sum of two consecutive even integers is 98.
21. The sum of three consecutive integers is less than 14.
22. The sum of three consecutive odd integers is no less than 15.
23. The sum of three consecutive multiples of 4 is greater than 16.
24. The sum of three consecutive even integers is not more than 42.
25. The sum of two consecutive integers is at least 25.
26. The sum of two consecutive multiples of 5 is at most 45.

Problems

Solve.

A

1. Find two consecutive integers whose sum is 383.
2. Find three consecutive integers whose sum is 66.
3. Find two consecutive odd integers whose sum is -160.
4. Find three consecutive even integers whose sum is -186.
5. Find three consecutive multiples of 7 whose sum is 84.
6. Find two consecutive multiples of 9 whose sum is -153.
7. Find three consecutive even integers such that the sum of the first and third integers is 192.
8. Find four consecutive odd integers such that the sum of the least integer and the greatest integer is 164.
9. Find the least two consecutive odd integers whose sum is more than 46.
10. Find the greatest two consecutive even integers whose sum is less than 180.
11. Three times an odd integer is eleven less than four times the next greater even integer. What is the odd integer?
12. The sum of an even integer and twice the next greater even integer is eight more than four times the greater integer. Find the lesser integer.
13. The lengths in meters of two sides of a rectangle are consecutive even integers. If ten more than half the length of the shorter side is added to twice the length of the longer side, the result is 59 m. Find the area of the rectangle.
14. The lengths in centimeters of two sides of a rectangle are consecutive odd integers. The perimeter is 120 cm. What is the area of the rectangle?

B 15. The present ages in years of four cousins are consecutive multiples of 3. Five years ago the sum of their ages was 46. Find their ages now.

16. The dimensions in meters of a rectangular field are consecutive multiples of 7. If each dimension were increased by 10 m, the perimeter of the field would be 306 m. Find the original dimensions of the field.

17. Find all sets of three consecutive integers whose sum is less than 24 if the sum of the least integer and twice the next greater integer is more than twice the greatest integer.

18. Find all sets of two consecutive even integers whose sum is no greater than 30 if five times the lesser of the two consecutive even integers is more than four times the greater.

19. Find all sets of three consecutive multiples of 3 whose sum is between -134 and -109.

20. Find all sets of three consecutive multiples of 6 whose sum is between -6 and 50.

21. The sum of an even integer and twice the next greater even integer is the same as the difference between five times the greater even integer and 14. Find the integers. *Hint*: There are two solutions.

22. Find all sets of three consecutive multiples of 11 for which the sum of the two lesser numbers is greater than 100 and the sum of the two greater numbers is less than 200.

C 23. Show that the sum of any two consecutive odd integers is even.

24. Show that the sum of any three consecutive odd integers is odd.

Computer Exercises

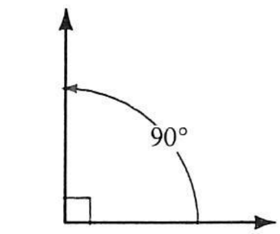
For students with computer experience

1. Write a program that will allow you to input any integer and will display whether the integer is even or odd.
2. Write a program that will compute the sum of ten consecutive integers when you input the least of the integers.
3. Modify the program that you wrote for Exercise 2 to compute the sum of ten consecutive *even* or *odd* integers when you input the least of the integers.
4. Modify the program that you wrote for Exercise 3 to compute the sum of *any number* of consecutive even or odd integers when you input the least integer and the number of integers to be added.
5. Write a program that will allow you to input a sum, S, and a number of consecutive integers, n, and will output n consecutive integers whose sum is S. Be careful to account for values that you may input for which there is no solution to the problem.

4–7 Problems about Angles

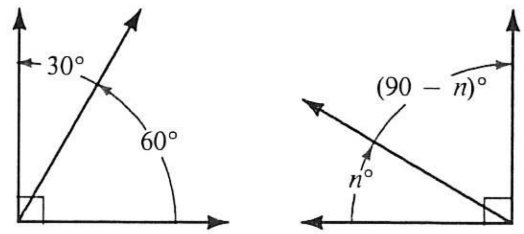
The facts shown in the chart that follows may be familiar to you. These facts can be used in solving certain problems about angles.

Right angle



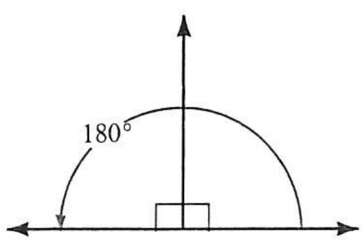
The measure of a **right angle** is 90°.

Complementary angles



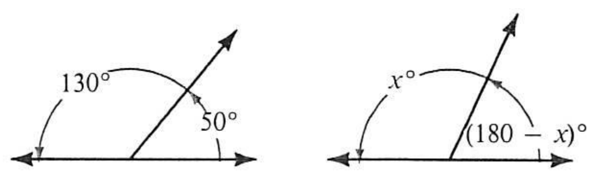
The sum of the measures of **complementary angles** is 90°.

Two adjacent right angles



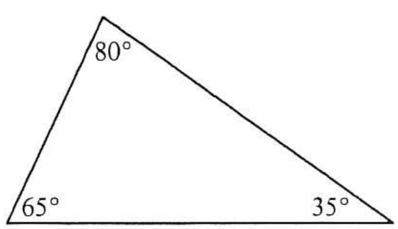
The sum of the measures of two right angles is 180°.

Supplementary angles



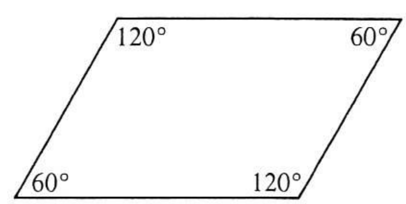
The sum of the measures of **supplementary angles** is 180°.

Triangle



The sum of the measures of the angles of a triangle is 180°.

Parallelogram



The opposite angles of a parallelogram are equal in measure.

EXAMPLE The measure of $\angle A$ (read "angle A") is $75°$ and the measure of $\angle B$ is $60°$.

a. Find the measure of the complement of $\angle A$.

b. Find the measure of the supplement of $\angle B$.

c. If $\angle A$ and $\angle B$ are two of the angles of a triangle, find the measure of the third angle.

SOLUTION a. The measure of $\angle A$ is $75°$.

$$90 - 75 = 15$$

$\therefore$ the measure of the complement of $\angle A$ is $15°$.

b. The measure of $\angle B$ is $60°$.

$$180 - 60 = 120$$

$\therefore$ the measure of the supplement of $\angle B$ is $120°$.

c. The sum of the measures of the three angles of a triangle is $180°$.

Let $x°$ = the measure of the third angle.

$$x + 75 + 60 = 180$$
$$x + 135 = 180$$
$$x = 45$$

$\therefore$ the measure of the third angle is $45°$.

Oral Exercises

State the measure of the angle that is the complement of the angle with the given measure.

1. $14°$	2. $71°$	3. $33°$	4. $52°$
5. $m°$	6. $5a°$	7. $(b + 2)°$	8. $(c - t)°$

State the measure of the angle that is the supplement of the angle with the given measure.

9. $125°$	10. $50°$	11. $10°$	12. $150°$
13. $n°$	14. $3r°$	15. $(e + 10)°$	16. $(2d - 40)°$

The measures of two angles of a triangle are given. State the measure of the third angle.

17. $20°, 30°$	18. $100°, 15°$	19. $15°, 70°$	20. $83°, 96°$
21. $x°, 5x°$	22. $c°, (17 - c)°$	23. $3b°, (25 - b)°$	24. $(5t + 47)°, (19 - 5t)°$

State an equation that represents the given information.

25. The measure of a certain angle is $40°$ less than the measure of its supplement.

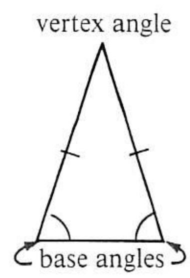
26. The measure of a certain angle is $10°$ more than the measure of its complement.

Problems

Solve.

A

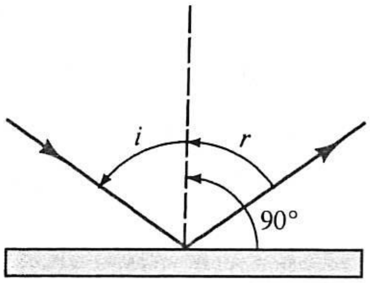
1. The measure of an angle is 58° more than the measure of its supplement. Find the measure of its supplement.
2. The measure of an angle is 26° less than the measure of its complement. Find the measure of the angle.
3. Find the measure of two complementary angles if the measure of one is 42° more than twice the measure of the other.
4. Find the measure of two supplementary angles if the measure of one is 24° more than three times the measure of the other.
5. Find the measure of an angle if the measure of its supplement is 4° more than twice the measure of its complement.
6. Find the measure of an angle if the sum of the measures of its complement and supplement is 162°.
7. The measure of one angle of a triangle is three times the measure of a second angle, and the measure of the third angle is 12° less than the sum of the measures of the other two. Find the measure of each angle.
8. The measure of one angle of a triangle is 20° more than the measure of the second angle. The measure of the third angle is 30° more than twice the sum of the measures of the first two angles. Find the measure of each angle in the triangle.
9. The sum of the measures of the four angles of a parallelogram is 360°. If the measure of one angle is 50°, find the measure of each of the other angles.
10. The sum of the measures of the four angles of a parallelogram is 360°. If the measure of one angle is 30° more than the measure of a second angle, find the measure of each of the four angles.
11. A triangle in which the three angles have equal measure is called an *equiangular triangle*. What is the degree measure of the complement of each angle of an equiangular triangle?
12. A triangle in which two angles, called base angles, have equal measure is called an *isosceles triangle*. In a certain isosceles triangle, the sum of the measures of the base angles is 88° less than the measure of the remaining angle. Find the measure of each angle.

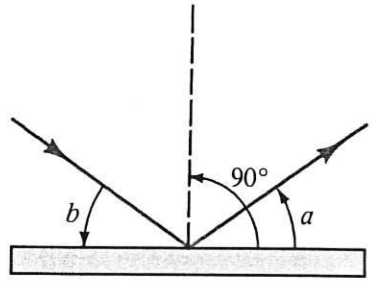


Ex. 12

13. The degree measures of two angles of a triangle are consecutive even integers. If the measure of the third angle is 22° more than the measure of the least angle of the triangle, what is the measure of each angle?
14. The degree measures of two angles of a triangle are consecutive multiples of 10. The measure of the third angle is 50° more than twice the measure of the least angle. Find the measure of each angle.

Exercises 15–18 refer to the Law of Reflection, $i = r$. That is, when a light ray strikes a reflecting surface, the measure, i, of the *angle of incidence* is equal to the measure, r, of the *angle of reflection.*





B **15.** $i = (3x - 10)°$
$r = (50 - x)°$
Find x.

16. $a = (5t + 5)°$
$b = (3t + 75)°$
Find t.

17. $i = (5m - 14)°$
$r = (7m - 86)°$
Find m.

18. $a = (18 - 4d)°$
$b = (113 - 9d)°$
Find d.

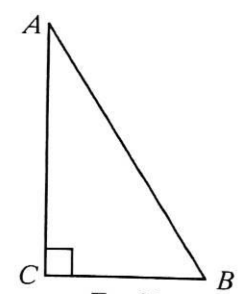
19. The measure of $\angle W$ is twice the measure of $\angle X$ and is 1° less than the measure of $\angle Y$. If the sum of the measures of the three angles is at least 56°, what is the least possible measure of $\angle W$?

20. The measure of $\angle R$ is three times the measure of $\angle S$. If the measure of $\angle R$ is at least 80° more than the measure of $\angle S$, what is the least possible measure of $\angle S$?

21. In $\triangle ABC$ (read "triangle A, B, C"), $\angle C$ is a right angle, and the measure of $\angle B$ is at least five times the measure of $\angle A$.

a. Find the least possible degree measure of $\angle B$.

b. Find the greatest possible degree measure of $\angle B$ if the measure of $\angle B$ is an integer.



Ex. 21

22. The measure of $\angle P$ is 40° less than the measure of $\angle D$ and is one half the measure of $\angle Q$. If the sum of the measures of the three angles is at most 140°, what is the greatest possible measure of $\angle P$?

C **23.** In $\triangle ABC$, the measure of $\angle B$ is three fifths the measure of the supplement of $\angle A$. The measure of $\angle C$ is four thirds the measure of the complement of $\angle B$. Find the degree measures of the three angles of the triangle.

24. The measure of an angle is at least one fourth the measure of its supplement and at most four times the measure of its complement. Find the possible degree measures of the angle.

4–8 Problems of Uniform Motion

An object that moves at a constant speed, or *rate*, is said to be in **uniform motion.** A formula that is often used in solving problems involving uniform motion is:

$$\text{rate} \cdot \text{time} = \text{distance}$$
$$r \cdot t = d$$

This formula may also be used for motion that is not uniform if the *average rate* is considered. To apply the formula correctly, the units used for the time and distance measurements must be the same as those used for the rate.

Three types of problems that can be solved using the formula $r \cdot t = d$ are illustrated in the examples that follow. The following standard symbols are used: s for seconds; min for minutes; h for hours.

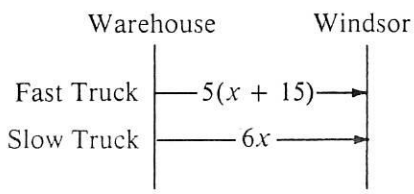
EXAMPLE 1 (Motion in the same direction)

Two delivery trucks heading for Windsor along the same route leave a warehouse at the same time. Their average speeds differ by 15 km/h. The faster truck reaches Windsor after 5 h of travel time. The slower truck takes an hour longer. Find the average speed of each truck.

SOLUTION

Step 1 The problem asks for the average speed of each truck.

Step 2 Let x = rate of slower truck in km/h.
Then $x + 15$ = rate of faster truck in km/h.



	rate (km/h)	time (h)	distance (km)
Faster truck	$x + 15$	5	$5(x + 15)$
Slower truck	x	6	$6x$

Step 3 $\underbrace{\text{Distance that faster truck went}}_{5(x + 15)} = \underbrace{\text{Distance that slower truck went}}_{6x}$

Step 4
$$5(x + 15) = 6x$$
$$5x + 75 = 6x$$
$$75 = x$$

Thus $x = 75$ and $x + 15 = 90$.

Step 5 Checking the results is left to you.
∴ the average speed of the slow truck is 75 km/h, and the average speed of the fast truck is 90 km/h.

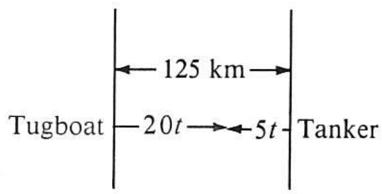
EXAMPLE 2 (Motion in opposite directions)

An oil tanker with engine trouble radios the mainland for a seagoing tugboat. At the time that the tugboat leaves the dock, the tanker is 125 km away and heading directly toward the dock. If the average speed of the tugboat is 20 km/h and that of the tanker is 5 km/h, how long will it take the two vessels to meet?

SOLUTION

Step 1 The problem asks for the number of hours the tugboat and tanker will travel before they meet.

Step 2 Let t = the number of hours it will take for the tugboat and tanker to meet.



	rate (km/h)	*time (h)*	*distance (km)*
Tugboat	20	t	$20t$
Tanker	5	t	$5t$

Step 3

$$\underbrace{\text{Distance traveled by tugboat}}_{20t} + \underbrace{\text{Distance traveled by tanker}}_{5t} = \underbrace{\text{Total distance traveled by tugboat and tanker}}_{125}$$

Step 4

$$20t + 5t = 125$$
$$25t = 125$$
$$t = 5$$

Step 5 Checking the result is left to you.

$\therefore$ the tugboat and tanker will meet in 5 h.

EXAMPLE 3 (Round trip)

Each day Kim jogs from her home to the lake and then walks back again along the same route. Her average speed is 5 m/s while jogging and 2 m/s while walking. If the whole trip takes 35 min, how far is it from Kim's home to the lake?

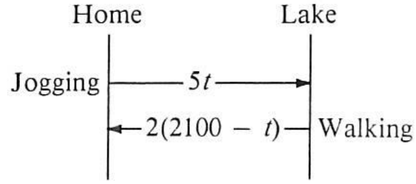
SOLUTION

Step 1 The problem asks for the distance from Kim's home to the lake.

Step 2 If you know the time it takes to go in one direction, you can find the distance. Since the rate is given in meters per second, the time will be found in seconds. Thus, the total time of 35 min must be changed into seconds.

Let t = time in seconds that it takes to jog to the lake.

Then $2100 - t$ = time in seconds that it takes to walk back.



	rate (m/s)	time (s)	distance (m)
Jogging	5	t	$5t$
Walking	2	$2100 - t$	$2(2100 - t)$

Step 3 $\underbrace{\text{Distance to lake}}_{5t} = \underbrace{\text{Distance back again}}_{2(2100 - t)}$

Step 4

$5t = 2(2100 - t)$
$5t = 4200 - 2t$
$7t = 4200$
$t = 600$

Since you know that it takes 600 s for Kim to jog to the lake at 5 m/s, you can find the distance from Kim's home to the lake.

$d = 5 \times 600$
$= 3000$

Step 5 Checking the result is left to you.
$\therefore$ the distance from Kim's home to the lake is 3000 m, or 3 km.

Oral Exercises

Let r = average rate, t = time, and d = distance. Find the quantity indicated using the given information. Give the correct units in each case.

1. $r = 12$ m/s, $t = 8$ s; d
2. $r = 15$ km/h, $d = 120$ km; t
3. $d = 300$ m, $t = 50$ min; r
4. $r = 90$ km/h, $t = 40$ min; d
5. $r = 100$ m/min, $d = 2$ km; t
6. $d = 80$ mi, $t = 2$ h 30 min; r

Solve.

7. Tom traveled 180 km in 3 h. What was his average speed?
8. Jane traveled at an average speed of 100 km/h. How far did she go in 90 min?
9. Alice drove a distance of 220 km at an average speed of 80 km/h. How long did the trip take?
10. Mike's home is 35 km from his office. On Tuesday, Mike leaves home at 8:00 A.M. At what time will he reach his office if he travels at an average speed of 50 km/h?

Problems

Solve.

A 1. The average speed of a moving van is 24 km/h faster than that of a delivery truck. If the van travels the same distance in 5 h as the delivery truck travels in 7 h, find the average speed of each vehicle.

2. Two airplanes start toward each other at the same time from airports located 1950 km apart. One plane flies at an average speed of 360 km/h. What should the average speed of the other plane be if they are to meet in 3 h?

3. Two campers leave Square Lake at the same time, one traveling west at an average rate of 75 km/h and the other traveling east at an average rate of 65 km/h. After how many hours will the campers be 490 km apart?

4. Lionel left Reedsville at 9:00 A.M. one day and drove to Fitzwilliam at an average rate of 80 km/h. At 10:00 A.M. the same day, Nancy left Reedsville and followed the same route. If both Lionel and Nancy arrived in Fitzwilliam at 3:00 P.M., at what average rate had Nancy traveled? How far is Reedsville from Fitzwilliam?

5. Reggie rode his tractor from his home to the agricultural exhibit and then walked back home along the same route. His average speed was 8 km/h while riding and 5 km/h while walking. How far is it from his home to the exhibit if his traveling time totaled 78 min?

6. Gina flew from Toronto to Chicago and back again. The average speed of the plane was 332 km/h to Chicago and 415 km/h back to Toronto. If the actual flying time for the round trip was 4.5 h, how far is it from Toronto to Chicago?

7. Becky and Rita are 648 m apart. Becky walks toward Rita at the rate of 2.25 m/s and Rita runs toward Becky. What is Rita's average speed if she reaches Becky in 1.6 min?

8. A passenger train traveling due west at 40 km/h passes a freight train traveling due east at 90 km/h. How long after they pass each other will the trains be 325 km apart?

9. Riding mostly downhill, Joan rode her ten-speed bicycle to the state fair in 1.5 h. It took 4.2 h to make the return trip, since her average speed was 36 km/h less than her average speed on the way to the fair. Find her average speed in each direction.

10. A commercial jet and a private airplane leave the same airfield at the same time and travel in opposite directions at average speeds of 880 km/h and 450 km/h, respectively. In how many minutes will they be 1463 km apart?

11. A helicopter traveling at an average speed of 225 km/h left San Jose 1 h after a train that had departed at 7:00 A.M. If the helicopter overtook the train in 0.8 h, find the average speed of the train.

12. Madeline made a certain trip by moped in 2 h. It took her 2.5 h to make the return trip, since her average speed returning was 12 km/h less than her speed going. Find Madeline's average speed in each direction.

13. Maria drove for two hours to get to the Fort William airport. She then flew from Fort William to Green Island. During the four hour flight to Green Island, the average speed of the plane was three times the average speed that Maria traveled on the drive to the airport. If Maria traveled a total distance of 1120 km, what was the average speed on each part of her trip?

14. An armored truck left the Federal Bank at noon. When a shipping error was discovered, an agent was sent by helicopter to overtake the truck. The helicopter left at 1:48 P.M. and flew at an average speed of 170 km/h. If the truck was overtaken at 3:24 P.M., what was the average speed of the truck?

B 15. A commercial plane had been flying for three hours when a change in the wind decreased the plane's average speed by 30 km/h. If the entire trip of 2540 km took 5 h, how far did the plane travel before the wind changed?

16. A private airplane had been flying for 1 h when a change of wind direction doubled the plane's average speed. If the entire trip of 384 km took 2.5 h, how far did the plane travel in the first hour?

17. A car traveled 462 km in 7 h. For the last 3 h of the trip its average speed was 20 km/h less than twice its average speed for the first 4 h. Find the two speeds at which the car traveled.

18. Calvin drove from Greenfield to Munsonville at an average speed of 64 km/h. By traveling at an average speed of 78 km/h, he could have arrived 10 min earlier. How far is it from Greenfield to Munsonville? Give your answer to the nearest tenth of a kilometer.

19. A cargo ship must travel at an average speed of 25 km/h to make its 14 h run on schedule. During the first 4 h, bad weather forced the captain to reduce speed to 20 km/h. What should the average speed of the ship be for the rest of the trip to keep on schedule?

20. A Great Lakes ore carrier must travel at an average speed of 24 km/h for 9 h to reach its destination on schedule. Fog forced the captain to reduce the speed to 21 km/h for the first 6 h. To arrive on time, what should be the ship's average speed for the rest of the trip?

21. In still water Meg can row 6 km/h. However, on the Branch River it took her 4 h to row upstream to her friend's cabin and only 2 h to return. Find the speed at which the river was flowing.

22. To go fishing, Art must row upstream for 3 h 20 min. The return trip only takes 1 h 40 min. If Art can row 4.5 km/h in still water, what is the speed at which the river is flowing? How far is it to the fishing location?

C

23. Ann lived in Bristol and Amy lived in Fairvale. At 2:00 P.M. they left their respective towns and walked in opposite directions, with Amy walking twice as fast as Ann. By 4:00 P.M., they were 66 km apart. If, instead, they had walked from the towns toward each other, they would have been 12 km apart at 3:00 P.M. How far apart are the towns?

24. Three jets depart from the airport at the same time. The average speed of the westbound plane is 50 km/h less than that of the southbound plane. The average speed of the eastbound plane is 60 km/h more than that of the southbound plane. The eastbound and westbound planes are 3150 km apart after 3 h. How far from the airport is the southbound plane at that time?

25. Two robots, T4-2 and B4-U, are programmed to roll toward each other, bump, return to their original positions, and then repeat the moves. T4-2 moves at 1.8 m/s and B4-U moves at 1.2 m/s. If eight seconds elapse between their first and second bumps, how far apart are their original positions?

26. Gerry and Tony live 11 km apart. Gerry leaves his home at 2:00 P.M. and bikes toward Tony's home at 25 km/h. At the same time, Tony leaves his home and bikes toward Gerry's home at 30 km/h. As soon as they reach each other's homes, they turn around and start back along the same route. They stop when they meet each other on the ride back to their own homes. How far are they from Gerry's home when they stop?

27. A freight train traveling 100 km/h took 48 s to pass a motorcyclist going in the same direction at 40 km/h. Assuming the length of the motorcycle is negligible, find the length of the train. If the train and the motorcycle were traveling in opposite directions, find to the nearest second how long they would have taken to pass each other completely.

28. The Midland Flyer, traveling at 90 km/h, overtakes another train that is traveling at 60 km/h in the same direction on a parallel track. The trains pass each other completely in 1.2 min. If the Midland Flyer is twice as long as the other train, what is the length of each train? If the trains were traveling in opposite directions, find to the nearest second how long they would have taken to pass each other completely.

PROGRAMMING IN BASIC

Often a computer program can be made more flexible by using INPUT statements. For example, the following program is a modification of the program on page 138. Here the INPUT statements allow you to change the dimensions of the figure without changing the lines of the program.

```
10 PRINT "TO CREATE GEOMETRIC"
20 PRINT "DESIGNS WITH ASTERISKS:"
30 PRINT
32 PRINT "GIVE A VALUE FOR M";
34 INPUT M
36 PRINT "GIVE A VALUE FOR N";
38 INPUT N
40 FOR I = 1 TO M
50 FOR J = 1 TO N
60 PRINT "*";
70 NEXT J
80 PRINT
90 NEXT I
100 END
```

Exercises

1. Type in the program as given. RUN the program for several values of M and N.

2. Delete lines 36 and 38.
   Change line 50 as follows: `50 FOR J = 1 TO I`
   LIST and RUN the revised program for several values of M.

3. Change line 50 as follows: `50 FOR J = 1 TO M + 1 - I`
   LIST and RUN the revised program for several values of M.

4. Change line 50 as follows: `50 FOR J = 1 TO 2 * I - 1`
   Then insert these lines:
   ```
   42 FOR J = 1 TO M + 1 - I
   44 PRINT " ";
   46 NEXT J
   ```
   LIST and RUN the revised program for several values of M.

Sometimes you can use the TAB function in a PRINT statement instead of using a loop that prints blank spaces. Note how TAB is used in the following exercises.

5. Delete lines 42, 44, and 46.
   Then insert this line: `45 PRINT TAB(M + 1 - I);`
   LIST and RUN the revised program for several values of M.

6. Change lines 40, 45, and 50 as follows:

```
40 FOR I = -M TO M
45 PRINT TAB(ABS(I) + 1);
50 FOR J = 1 TO 2 * M - 2 * ABS(I) + 1
```

LIST and RUN the revised program for several values of M.

7. To explore further how TAB works, type in and RUN this program.

```
NEW
10 FOR I = 1 TO 6
20 PRINT " ";
30 NEXT I
40 PRINT "6"
50 PRINT TAB(6);"6"
60 END
```

If the "6" printed in line 50 aligns with the "6" printed in line 40, the TAB on the computer you are using *skips over* 6 spaces. If the numbers do *not* align, consult the computer manual for a description of its TAB function.

4–9 Mixture Problems

A merchant often mixes goods of two or more kinds in order to sell a blend at a given price. Similarly, a chemist mixes solutions of different strengths of a chemical to obtain a solution of a desired strength. Problems related to these situations are often called *mixture problems.*

EXAMPLE 1 Find the number of kilograms of nuts worth $4.00 per kilogram and the number of kilograms of raisins worth $5.20 per kilogram that should be mixed to produce 3 kg of a snack worth $4.52 per kilogram.

SOLUTION

Step 1 The problem asks for the number of kilograms of nuts and the number of kilograms of raisins to use in 3 kg of a mixture.

Step 2 Let n = the number of kilograms of nuts.
Then $3 - n$ = the number of kilograms of raisins.

	Number of kg	*Value per kg*	*Total value*
Nuts	n	4	$4n$
Raisins	$3 - n$	5.2	$5.2(3 - n)$
Mixture	3	4.52	4.52(3)

Step 3 The sum of the values of the original ingredients must equal the value of the mixture.

$$\underbrace{\text{Value of nuts}}_{4n} + \underbrace{\text{Value of raisins}}_{5.2(3-n)} = \underbrace{\text{Value of mixture}}_{4.52(3)}$$

Step 4

$$\begin{aligned} 4n + 5.2(3 - n) &= 4.52(3) \\ 4n + 15.6 - 5.2n &= 13.56 \\ -1.2n &= -2.04 \\ n &= 1.7 \\ 3 - n &= 1.3 \end{aligned}$$

Step 5 Checking the results is left to you.

∴ 1.7 kg of nuts and 1.3 kg of raisins are needed.

EXAMPLE 2 A scientist has 200 mL of a solution that is 40% acid. How many milliliters of water should be added to make a solution that is 25% acid?

SOLUTION

Step 1 The problem asks for the number of milliliters of water that should be added to make a solution that is 25% acid.

Step 2 Let x = number of milliliters of water to be added.

	Volume of solution	*% acid*	*Volume of acid*
Original solution	200	40	0.40(200)
Added water	x	0	0
New solution	$200 + x$	25	$0.25(200 + x)$

Step 3

$$\underbrace{\text{Original volume of acid}}_{0.40(200)} + \underbrace{\text{Added volume of acid}}_{0} = \underbrace{\text{Total volume of acid}}_{0.25(200 + x)}$$

Step 4

$$\begin{aligned} 0.40(200) + 0 &= 0.25(200 + x) \\ 80 &= 50 + 0.25x \\ 30 &= 0.25x \\ x &= 120 \end{aligned}$$

Step 5 Checking the results is left to you.

∴ 120 mL of water should be added.

Many problems that appear to be unrelated to each other can be thought of as "mixture" problems. If you can identify a problem as a mixture problem, then you can use the techniques of this section in solving it. All the problems in the following exercises can be treated as mixture problems.

Oral Exercises

1. Eric wishes to mix together some common stone worth $.10 per kilogram and some granite chips worth $.15 per kilogram to produce a decorative garden mixture. How much of each kind of stone should be mixed to obtain 300 kg of a mixture that is worth $.13 per kilogram?

   a. Copy and complete the chart.

	Number of kg	Value per kg	Total value
Common stone	x	?	?
Granite chips	?	?	?
Mixture	?	?	?

   b. State an equation that can be used to solve the problem.

   c. Solve the problem.

2. Lucy has a collection of dimes and quarters. If she has 80 coins in all and their total value is $12.20, how many coins are there of each type?

   a. Copy and complete the chart. Note that one space is left blank since it is not necessary to find the "average value" of each coin.

	Number of coins	Value per coin	Total value
Dimes	x	?	?
Quarters	?	?	?
Mixture of Coins	?		?

   b. State an equation that can be used to solve the problem.

   c. Solve the problem.

Problems

Use a chart in solving each of the following "mixture" problems.

A

1. How much gold valued at \$12 per gram must be mixed with silver valued at \$.80 per gram to obtain 20 g of an alloy worth \$5 per gram?
2. How many liters of orange juice worth \$1.80 per liter should be mixed with 2.4 L of lemonade worth \$1.15 per liter to produce a punch worth \$1.54 per liter?
3. A 400 g solution is 25% salt. How much salt must be added to produce a solution that is 40% salt?
4. A 30 L solution is 80% antifreeze. How much water must be added to produce a solution that is 60% antifreeze?
5. Matt has twice as many quarters as dimes in his coin collection. If the dimes and quarters together total \$9.00, how many of each kind of coin does he have?
6. Cecelia bought some 22¢ and some 17¢ stamps for the school office. Altogether she purchased 37 stamps for \$7.54. How many stamps of each kind did she buy?
7. Bus fares from Amherst to Mount Mohawk are \$26 for adults and \$18 for students. How many students are on the bus if a total of \$1668 was collected from the 70 passengers?
8. The registration fee for a whale watching trip was \$20 for nonmembers and \$15 for members. If \$15,950 was collected from 830 people, how many of the people who registered were members?
9. Jane took three hours to drive to her friend's home on the weekend. Her average speed was 85 km/h for the first two hours. What was her average speed for the last hour of the trip if her average speed was 75 km/h for the whole trip?
10. For the first hour of a four hour trip, the average speed of a train was 40 km/h. If the average speed for the whole trip was 70 km/h, what was the average speed for the last three hours of the trip?

Solve.

B

11. Shirley has a quantity of yellow tulip bulbs worth 10¢ each, pink tulip bulbs worth 20¢ each, and red tulip bulbs worth 25¢ each. In her total collection of bulbs, there are five more pink bulbs than yellow bulbs and twice as many red bulbs as the sum of the other two kinds. If the bulbs altogether are worth \$133.50, how many of each kind does Shirley have?
12. Kevin has \$6.45 in coins in his cash box. The number of quarters is one less than twice the number of dimes. The number of nickels is one less than twice the number of quarters. The value of the pennies is the same as the value of the nickels. How many of each type of coin does he have?

13. The average mark on a test in an algebra class is 80. If the two lowest scores of 34 and 48 are not counted, the remaining scores would average 83. How many students are in the algebra class?

14. At a family reunion, the average age of all those present was 45 years. If the two oldest people, aged 86 and 84 years, had not been present, the average age would have been 41 years. How many people were at the reunion?

C 15. Dominique flew her light plane from Buffalo to Cleveland at an average speed of 150 km/h. She waited at Cleveland one half the time that she had been in the air on her trip to Cleveland. Returning home, she took a route that was 60 km longer and flew at an average speed of 180 km/h. If the total trip took five hours, how long did she stay in Cleveland?

16. Stan wants to fill a 204 L drum with a mixture of oil and gasoline so that the ratio of oil to gasoline is 1 to 16. He bought oil at $.85 per liter and gasoline at $.34 per liter. What is the minimum he can charge per liter if he wants to make at least 20% profit? Give the answer in a whole number of cents.

17. A grass seed mixture is 30% rye seed and 10% bluegrass seed. How many kilograms of seed that contains neither bluegrass nor rye seed should be added to 80 kg of the mix to produce a blend that is 20% rye seed? What percent of the new blend will be bluegrass seed?

18. A 22 kg solution of salt and water is 24% salt. Water is evaporated from this solution to produce a solution that is 32% salt. If 2.2 kg of salt is then added to the solution that is 32% salt, what percent of this new solution will be salt?

Computer Exercises For students with computer experience

1. Write a program that will allow you to input values for x, y, z, and w and will compute the percent acid of the solution that results when x mL of a solution that is y% acid is added to z mL of a solution that is w% acid.

2. Write a program that will allow you to input values for x, y, and z and will compute the percent acid of the solution that results when x mL of water is added to y mL of a solution that is z% acid.

3. Write a program that will allow you to input values for x, y, and z and will compute the number of mL of water that must be added to x mL of a solution that is y% acid to obtain a solution that is z% acid.

4. Modify the program that you wrote for Exercise 3 so that it will allow you to input values for x, y, z, and w and will compute the number of mL of a solution that is w% acid that must be added to x mL of a solution that is y% acid to obtain a solution that is z% acid.

4–10 Problems without Solutions

Not every problem has a solution. Consider this example.

EXAMPLE 1 Find three consecutive odd integers whose sum is 144.

SOLUTION

Step 1 The problem asks for three consecutive odd integers whose sum is 144.

Step 2 Let n = first odd integer. Then:
$n + 2$ = next greater odd integer,
$n + 4$ = greatest consecutive odd integer

Step 3 The sum of the three consecutive odd integers is 144.

$$n + (n + 2) + (n + 4) = 144$$

Step 4

$$\begin{aligned} n + (n + 2) + (n + 4) &= 144 \\ 3n + 6 &= 144 \\ 3n &= 138 \\ n &= 46 \end{aligned}$$

Step 5 The numbers obtained are 46, 48, and 50. However, these are consecutive even integers, not consecutive odd integers. Therefore they do not satisfy the conditions in the problem.

This example is an illustration of a problem in which the conditions given are *inconsistent*. This means that all the conditions in the problem cannot be true at the same time. In fact, you can prove that the sum of three consecutive odd integers is always odd. (See Exercise 24 on page 183.) Therefore, such a sum can never equal 144.

In reading problems, you should be on the lookout for inconsistent conditions. You should also be able to recognize problems in which not enough facts are given for you to obtain a definite answer.

EXAMPLE 2 The sum of two numbers is 10. Find the numbers.

SOLUTION There is an infinite set of pairs of numbers whose sum is 10. For example:

$$1 + 9 = 10, \quad 2 + 8 = 10, \quad 3\tfrac{1}{2} + 6\tfrac{1}{2} = 10, \quad 5.25 + 4.75 = 10,$$

and so on. In order to determine the required numbers, additional information is needed.

$\therefore$ the solution cannot be determined.

Some of the problems in the following set can be solved. Others fail to have solutions either because their conditions are inconsistent or because too few facts are given.

Problems

Solve. If a problem has no solution, explain why.

A

1. Find two consecutive integers whose sum is 102.
2. Find the least two consecutive integers whose difference is 54.
3. The lengths in meters of the sides of a triangle are three consecutive even integers. If the perimeter of the triangle is 132 m, find the length of each side.
4. The lengths in meters of two adjacent sides of a rectangle are consecutive multiples of 5. If the perimeter of the rectangle is 125 m, find the dimensions.
5. In $\triangle ABC$, the measure of $\angle B$ is three more than four times the measure of $\angle A$. If $\angle B$ and $\angle C$ are complementary angles, find the measure of each angle.
6. $\angle A$ and $\angle B$ are complementary angles, each with measure between $0°$ and $90°$. The sum of twice the measure of $\angle A$ and three times the measure of $\angle B$ is $180°$. Find the measures of $\angle A$ and $\angle B$.
7. A clerk took a twenty-dollar bill to the bank to get change. The clerk asked for twice as many dimes as nickels and three times as many quarters as nickels. Was the teller able to fulfill the request?
8. The difference between a two-digit number and the number obtained by reversing its digits is 9. If the digits are consecutive integers, find the number.
9. Tom has 18 coins. Some are dimes and the rest are nickels. What is the greatest number of dimes he can have if the value of the nickels is greater than that of the dimes?
10. A total of \$458 was collected for admission from 132 people who attended a show. If adults paid \$4.00 each and children paid \$2.50 each, how many children attended the show?
11. On a trip of 210 km, Gary traveled by train for 3 h and by bus for the rest of the trip. The average speed of the train was 15 km/h more than that of the bus. Find the average speed of the bus.
12. Susan walks at an average speed of 2.5 m/s to her school, which is 3 km from her home. Fifteen minutes after Susan left for school, her brother discovered she had forgotten her lunch. How fast must he ride his bicycle in order to overtake her before she reaches school?

B

13. A freight train takes 16 h to travel from Moncton to Parkroyal. A passenger train makes the trip in 12 h. If the average speed of the passenger train is 30 km/h more than the freight train, how far is it from Moncton to Parkroyal?
14. Two cars leave a rest stop at the same time and travel south on the same road. Five hours later the slower car passes a service plaza that the faster car had passed an hour earlier. If their average speeds differ by 17 km/h, how far is the service plaza from the rest stop?

15. Eighteen-carat gold contains 18 parts by mass of gold to 6 parts of other metals. Fourteen-carat gold contains 14 parts of gold and 10 parts of other metals. How many kilograms of eighteen-carat gold should be mixed with fourteen-carat gold to obtain 60 kg of an alloy containing 17 parts gold and 7 parts of other metals?

16. Terry's average grade on the first six tests in her history class was 75%. She got 100% on each of the next two tests. If the highest score possible on any test is 100%, what grades must Terry get on the final two tests in order to have an average of 90% for the course?

C 17. Lisa travels at 30 km/h on the way to visit a friend who lives 90 km away. How fast must she go on the return trip if she wants her average speed for the whole trip to be 40 km/h? 50 km/h? 60 km/h?

18. Find the least two positive integers whose sum is an even integer and whose difference is an odd integer.

Self-Test 2

VOCABULARY

consecutive integers (p. 179)
multiple (p. 180)
consecutive multiples (p. 180)
consecutive even integers (p. 181)
consecutive odd integers (p. 181)
right angle (p. 184)
complementary angles (p. 184)
supplementary angles (p. 184)
uniform motion (p. 189)

Solve.

1. Find three consecutive even integers whose sum is 154 more than the greatest of these three integers. *Obj. 1, p. 179*
2. In $\triangle ABC$, the measure of $\angle A$ is 12° less than the measure of $\angle B$ and the measure of $\angle C$ is 18° more than the measure of $\angle A$. Find the measure of each angle.
3. Ed's average speed is 20 km/h when he is jogging. He leaves his school at 10:00 A.M. and starts jogging to the next town. His coach leaves at 10:15 A.M. in his car and follows the same route. If the coach's average speed is 60 km/h, how far from the school will the coach overtake Ed?
4. How many kilograms of dried apricots worth $8.30 per kilogram should be mixed with dried apples worth $6.50 per kilogram to produce 12 kg of a mixture worth $6.95 per kilogram?
5. Sally has twenty coins in her purse. Some of them are dimes and the remainder are quarters. If she does not have enough money to buy a book for $3.98, what is the most number of quarters she can have? *Obj. 2, p. 179*

Check your answers with those at the back of the book.

EXTRA

Symbolic Logic: Boolean Algebra

Most people think of algebra as the study of operations with numbers and variables. Another kind of algebra, which is used in the design of electronic digital computers, involves operations with logical statements. This "algebra of logic" is called **Boolean algebra.** (Boolean algebra is named after George Boole, a nineteenth-century British mathematician who developed the fundamental principles on which this algebra is based.)

In Boolean algebra you use letters such as p, q, r, s, and so on, to represent statements. For example, you might let p represent the statement "3 is an odd integer," and q, the statement "5 is less than 3." In this case, the statement p has the truth value T (the statement is true), whereas q has the truth value F (the statement is false).

The following table shows the operations that are used in Boolean algebra to produce compound statements from any given statements p and q.

Operation	*How Read*	*Symbols*
conjunction	p and q	$p \wedge q$
disjunction	p or q	$p \vee q$
conditional	If p, then q	$p \rightarrow q$
equivalence	p if and only if q	$p \leftrightarrow q$
negation	not p	$\sim p$

The rules for assigning truth values to compound statements are shown in the five *truth tables* that follow.

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Notice that the conjunction $p \wedge q$ is true when *both* p and q are true; otherwise, $p \wedge q$ is false. On the other hand, the disjunction $p \vee q$ has the value T provided *at least one* of the statements p, q is true.

Conditional

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Equivalence

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

When is the conditional $p \to q$ false? Only when p is true and q is false.

The equivalence $p \leftrightarrow q$ is a brief way of stating the conjunction

$$(p \to q) \wedge (q \to p).$$

The equivalence is true when p and q are both true or both false.

Negation

p	$\sim p$
T	F
F	T

The negation $\sim p$ is the denial of p. Therefore, it is reasonable to agree that $\sim p$ is false when p is true, and true when p is false.

EXAMPLE 1 Let r represent "$1 > 2$," and s, "$2 < 4$." Read each of the following statements. Then, referring to the preceding truth tables, give the truth value of the statement and a reason for the answer.

a. $r \wedge s$ **b.** $r \vee s$ **c.** $r \to s$ **d.** $r \leftrightarrow s$ **e.** $\sim r$

SOLUTION **a.** $r \wedge s$: $1 > 2$ *and* $2 < 4$.
F, because the truth value of r is F.

b. $r \vee s$: $1 > 2$ *or* $2 < 4$.
T, because the truth value of s is T.

c. $r \to s$: *If* $1 > 2$, *then* $2 < 4$.
T, because the truth value of r is F, and that of s is T.

d. $r \leftrightarrow s$: $1 > 2$ *if and only if* $2 < 4$.
F, because r and s have different truth values.

e. $\sim r$: *Not* $(1 > 2)$, that is, $1 \leq 2$.
T, because the truth value of r is F.

EXAMPLE 2 Show that for all truth values of r, p, and q, the truth value of

$$r \vee (p \wedge q) \leftrightarrow (r \vee p) \wedge (r \vee q)$$

is T.

SOLUTION Construct a truth table containing the eight combinations of truth values of r, p, and q.

Can you tell how the table on the following page was constructed? Entries in column 4 were obtained by "*and*-ing" the entries in columns 2 and 3. The entries in column 5 result from "*or*-ing" the entries in columns 1 and 4. The rest of the table is obtained similarly.

r	p	q	$p \wedge q$	$r \vee (p \wedge q)$	$r \vee p$	$r \vee q$	$(r \vee p) \wedge (r \vee q)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

When you compare the fifth column of this table to the last column, you can see that $r \vee (p \wedge q)$ and $(r \vee p) \wedge (r \vee q)$ have the same array of truth values. Therefore,

$$r \vee (p \wedge q) \leftrightarrow (r \vee p) \wedge (r \vee q)$$

is a true statement no matter what truth values are assigned to r, p, and q.

A compound statement that is true for all truth values of its component statements is called a **tautology**.

Exercises

Assume that r and p are true statements and that q is a false statement. Determine the truth value of each statement.

1. $q \rightarrow r$
2. $\sim r \wedge p$
3. $p \vee \sim q$
4. $r \wedge (p \vee q)$
5. $p \vee (q \wedge r)$
6. $(p \vee r) \rightarrow q$
7. $\sim p \rightarrow (q \vee \sim r)$
8. $r \rightarrow (q \rightarrow r)$
9. $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$

Given that $p \rightarrow q$ is a false statement, show that each statement is true.

10. $q \rightarrow p$
11. $p \wedge \sim q$
12. $(p \vee q) \wedge p$

Construct a truth table for each of the following statements. Then tell whether or not each statement is a tautology.

13. $(p \vee q) \rightarrow p$
14. $(q \wedge \sim q) \rightarrow p$
15. $(p \vee q) \leftrightarrow (q \vee p)$
16. $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
17. $r \wedge (p \vee q) \rightarrow (r \wedge p) \vee (r \wedge q)$
18. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Chapter Summary

Statements 1–4 are true for all values of each variable except as noted.

1. *Axiom of comparison* One and only one of the following statements is true: $a < b$, $a = b$, or $b < a$

2. *Transitive property of order*
   a. If $a < b$ and $b < c$, then $a < c$.
   b. If $a > b$ and $b > c$, then $a > c$.

3. *Addition property of order*
   a. If $a < b$, then $a + c < b + c$ and $c + a < c + b$.
   b. If $a > b$, then $a + c > b + c$ and $c + a > c + b$.

4. *Multiplication property of order*
   a. If $a < b$ and $c > 0$, then $ac < bc$ and $ca < cb$.
   b. If $a > b$ and $c > 0$, then $ac > bc$ and $ca > cb$.
   c. If $a < b$ and $c < 0$, then $ac > bc$ and $ca > cb$.
   d. If $a > b$ and $c < 0$, then $ac < bc$ and $ca < cb$.

5. Inequalities that have the same solution set over a given domain are called *equivalent inequalities*. Each of the following transformations of a given inequality will produce an equivalent inequality.
   1. Substituting for either side of the inequality an expression equivalent to that side.
   2. Adding to (or subtracting from) each side the same real number.
   3. Multiplying (or dividing) each side by the same positive number.
   4. Multiplying (or dividing) each side by the same negative number and reversing the order of the inequality.

6. If S and T are any sets, the set consisting of the members belonging to both S and T is called the *intersection* of S and T and is denoted by $S \cap T$. The set consisting of the members belonging to *at least one* of the sets S and T is the *union* of S and T and is denoted by $S \cup T$.
7. Sets having no members in common are *disjoint sets*.
8. A sentence that is formed by joining two sentences with the word *and* is called a *conjunction*. For a conjunction to be true, *both* of the joined sentences must be true. A sentence that is formed by joining two sentences with the word *or* is a *disjunction*. For a disjunction to be true, *at least one* of the joined sentences must be true.
9. Some open sentences involving absolute value can be expressed as conjunctions while others can be expressed as disjunctions.
10. Problems involving integers, angles, uniform motion, and mixtures can often be solved by following the plan outlined on page 132.
11. Some problems cannot be solved either because of insufficient information or because the information given is inconsistent.

Chapter Review

Write the letter of the correct answer.

1. If $b < c$, which of the following statements is false? *4-1*
   a. $b - 5 < c - 5$ b. $5 + b < 5 + c$
   c. $\frac{b}{-5} < \frac{c}{-5}$ d. $5b < 5c$

2. Solve $10a + 4 > 36 + 2a$. *4-2*
   a. $\{a: a > 5\}$ b. $\{a: a < 4\}$ c. $\{a: a < 5\}$ d. $\{a: a > 4\}$

3. Solve $5z + 3 - 7z \leq 11$.
   a. $\{z: z \geq -4\}$ b. $\{z: z \leq -4\}$ c. $\{z: z \leq -7\}$ d. $\{z: z \geq -7\}$

4. Specify the union of {1, 2, 3} and {2, 3, 4}. *4-3*
   a. {2, 3} b. {1, 4} c. {1, 2, 3, 4} d. Ø

5. Specify the intersection of {the real numbers between −6 and 2} and {the real numbers greater than −1}.
   a. {the real numbers between −6 and −1}
   b. {the real numbers greater than −6}
   c. {the real numbers between −1 and 2}
   d. Ø

Solve.

6. $-6 < x - 2 < 4$ *4-4*
   a. $\{x: -8 < x < 2\}$ b. $\{x: -4 < x < 2\}$
   c. $\{x: -4 < x < 6\}$ d. $\{x: -8 < x < 6\}$

7. $2y + 3 \leq -2$ or $3y - 5 > 4$
   a. $\{y: y \leq -\frac{5}{2}$ or $y > 3\}$ b. $\{y: -\frac{5}{2} \leq y < 3\}$
   c. $\{y: y \geq -\frac{5}{2}$ and $y < 3\}$ d. $\{y: y < -3$ or $y \geq \frac{5}{2}\}$

8. $5 + |2m - 9| \leq 6$ *4-5*
   a. $\{m: m \leq 5\}$ b. $\{m: m \leq 4\}$
   c. $\{m: 4 \leq m \leq 5\}$ d. $\{m: -5 \leq m \leq -4\}$

9. $|1 - 3n| > 7$
   a. $\{n: -\frac{8}{3} < n < 2\}$ b. $\{n: n < -2$ or $n > \frac{8}{3}\}$
   c. $\{n: -2 < n < \frac{8}{3}\}$ d. $\{n: n < -\frac{8}{3}$ or $n > 2\}$

10. Find the least three consecutive integers whose sum is greater than −87. *4-6*
    a. −29, −28, −27 b. −31, −32, −33
    c. 31, 32, 33 d. −29, −30, −31

11. The measure of one angle of a triangle is 1° less than twice the measure of the second angle, and the measure of the third angle is 21° more than twice the sum of the measures of the other two. Find the measure of the third angle. *4-7*

a. 18° b. 127° c. 35° d. 106°

12. A Coast Guard helicopter flew toward a ship that was heading for shore with an injured sailor. The average speed of the ship was 40 km/h and the average speed of the helicopter was 300 km/h. The helicopter started from the shore when the ship was 510 km away. How long did it take them to meet? *4-8*

a. 0.5 h b. 1 h c. 1.75 h d. 1.5 h

13. A 60 L solution is 60% acid. How many liters of water must be added to produce a solution that is 45% acid? *4-9*

a. 18 b. 20 c. 24 d. 32

14. Tell why this problem has no solution: "Find four consecutive odd integers whose sum is −60." *4-10*

a. The sum of four odd integers cannot be an even integer.
b. The sum of four odd integers cannot be negative.
c. There are not enough facts given to write an equation.
d. The only solution of an equation that repesents the relationships in the problem is an even integer.

Chapter Test

1. Name the property that is illustrated by the following: "If $-3m > 15$, then $m < -5$." *4-1*

Solve.

2. $7 - 2p > 15$ 3. $2q + 50 < 7q + 2 + 3q$ *4-2*

Let A = {the natural numbers less than 5} and B = {the integers greater than −3}. Specify the following.

4. $A \cup B$ 5. $A \cap B$ *4-3*

Solve.

6. $-8 \leq 1 - 3x < 10$ 7. $3y + 5 < 3$ or $2y - 5 > 3$ *4-4*

8. $|3 - 2m| \geq 5$ 9. $2|4 + n| - 5 < 9$ *4-5*

10. Find the greatest two consecutive odd integers whose sum is less than 85. *4-6*

11. The measure of an angle is 16° more than the measure of its supplement. Find the measure of the angle. 4-7

12. Jim's average speed is 8 km/h while walking and 12 km/h while jogging. It takes him 0.5 h longer to walk from the school to the pool than it takes him to jog the same distance. How far is it from the school to the pool? 4-8

13. How many liters of water must be added to 90 L of a solution that is 80% antifreeze to obtain a solution that is 60% antifreeze? 4-9

14. Explain why the following problem has no solution: The degree measures of two angles of a triangle are consecutive even integers. The measure of the third angle is 37° more than the measure of the smallest angle. Find the measure of each angle. 4-10

Mixed Review

Simplify.

1. $\dfrac{16 - 4(2)}{12 - 3(3)}$
2. $\dfrac{3^2 + 5^2}{(5 - 3)^2}$
3. $-\frac{2}{3} \div \frac{4}{9}$
4. $-\frac{2}{7} - \frac{5}{7} + \frac{4}{7}$
5. $-4a^2 + a^2$
6. $(-5b)(-2b)$
7. $-12s^2 \div (-3)$
8. $3t - 5t^2 - t$
9. $-(-5 + 3) + (-8)$
10. $24 \div 3(7 - 3) + 4$
11. $15 + 2(3^2 - 2)^2$
12. $9m^2 - 4(m^2 + 2)$
13. $3(2n - 1) - 2(3n + 1)$
14. $(4x^2 - 2x) - (x^2 - 5)$

Graph the solution set of each open sentence on a number line.

15. $0 \le x < 8$
16. $y < 2$ or $y \ge 6$
17. $|z| < 2$
18. $|a| > -3$
19. $b > -2$ and $b < 3$
20. $2 < |c| < 5$

Solve.

21. $\frac{1}{a} = -2$
22. $-\frac{b}{3} = \frac{1}{6}$
23. $4 - 2x = 6$
24. $4y > 5y + 1$
25. $5 - (2 - m) < 6$
26. $3n - 4(n - 1) = 10$
27. $|p| < -2$
28. $-q > 7$
29. $|x - 5| = 2$
30. $|y + 1| + |-7| = 4$
31. $-9 < 2j - 1 < 2$
32. $-2k < 8$ or $-3k > 9$
33. Jane has six more nickels than quarters and two fewer dimes than nickels. If she has a total of $2.70, how many of each type of coin does she have?
34. Find three consecutive odd integers whose sum is -162.
35. Three years from now, Bob will be three times as old as he was three years ago. How old is Bob now?
36. How many liters of water must be added to 20 L of a solution that is 45% alcohol to obtain a solution that is 30% alcohol?

PREPARING FOR COLLEGE ENTRANCE EXAMS

Strategy for Success: Before you actually answer any questions, you may find it helpful to skim an entire section of an exam in order to get an *overview* of the questions. You may wish to answer the easiest questions first, then proceed to the harder ones. Do not take time to double-check your answers unless you finish all the questions before the deadline and have extra time.

Decide which is the best of the choices given and write the corresponding letter on your answer sheet.

1. Under which conditions is $x + y = |x| - |y|$ a true statement?
I. $x > 0$ and $y \leq 0$ II. $x < 0$ and $y \geq 0$ III. $x > 0$ and $y \geq 0$
(A) I only **(B)** II only **(C)** III only **(D)** I and II only **(E)** I, II, and III

2. Solve $x + 3 \leq -4$ or $-2x < 14$.
(A) $\emptyset$ **(B)** $\mathcal{R}$ **(C)** $\{x: x \geq 7\}$ **(D)** $\{x: x \leq -7\}$ **(E)** $\{x: -7 < x \leq 7\}$

3. Find the least of four consecutive even integers such that the third integer is equal to the sum of the first integer and twice the fourth integer.
(A) 2 **(B)** -4 **(C)** -2 **(D)** 18 **(E)** -12

4. Which of the following sets are closed under division, excluding division by zero?
I. $\{-3, 0, 3\}$ II. $\{-3, -1, 1, 3\}$ III. {the positive real numbers}
(A) I only **(B)** II only **(C)** III only **(D)** I and III only **(E)** II and III only

5. Solve $\frac{1}{5}|-x| = 10$.
(A) $\{-2\}$ **(B)** $\{-50\}$ **(C)** $\{2, -2\}$ **(D)** $\{50, -50\}$ **(E)** $\emptyset$

6. Solve for m in the equation $b = \frac{1}{2}mr^2$.
(A) $m = \frac{b}{2r^2}$ **(B)** $m = \frac{2b}{r^2}$ **(C)** $m = \frac{r^2}{2b}$ **(D)** $m = \frac{2r^2}{b}$ **(E)** $m = \frac{2m}{r^2}$

7. Carla's monthly salary for part of the year was \$1015. After she received a raise, her monthly salary for the remainder of the year was \$1145. Her total earnings for the year, including a bonus of \$1500, were \$14,720. For how many months did she work at the lesser salary?
(A) 4 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 10

8. Which of the following is equivalent to $\frac{1}{2}x + \frac{1}{3} = 4$?
I. $\frac{1}{2}x = \frac{11}{3}$ II. $x + \frac{2}{3} = 8$ III. $3x + 2 = 24$
(A) I only **(B)** II only **(C)** III only **(D)** I and II only **(E)** I, II, and III

APPLICATION

Acceleration

It is important for an automobile driver to be able to change the rate at which an automobile is moving. When the rate of motion, or speed, of a car increases, the car is said to *accelerate*.

In physics, acceleration is defined in terms of *velocity* rather than speed. A **velocity** specifies the direction of travel as well as the speed. For example, it is more informative to know that a car is traveling at a velocity of 40 km/h *due north* than to know only that its speed is 40 km/h.

The **acceleration** of an object can be defined as the rate at which its velocity changes over a given period of time. That is:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

Since velocity involves both direction and speed, a change in direction produces a change in acceleration whether or not a change in speed is also involved. For example, when you drive along a curved road at a constant speed, the direction of motion, and thus the acceleration, is changing.

The simplest case of acceleration to analyze involves an object that is moving in a straight line at a constant acceleration. The acceleration in this case can be expressed mathematically as

$$a = \frac{v_f - v_0}{t},$$

where a = constant acceleration, v_0 = initial velocity, v_f = final velocity, and t = time.

Suppose that a car is traveling along a straight road at a constant velocity of 25 km/h. The car then begins to accelerate at a constant rate. After 5 s of constant acceleration, the car is traveling at a velocity of 50 km/h. What is the acceleration of the car over the 5 s period?

Since $v_0 = 25$ km/h, $v_f = 50$ km/h, and $t = 5$ s,

$$a = \frac{v_f - v_0}{t} = \frac{50\text{ km/h} - 25\text{ km/h}}{5\text{ s}} = 5\text{ km/h/s}.$$

This acceleration is read as "five kilometers per hour per second" and means that the velocity of the car increases by 5 km/h each second.

Now suppose that a car starts from rest, that is, $v_0 = 0$. It then accelerates at a constant rate of 4 km/h/s along a straight road. At what velocity will it be traveling at the end of 15 s?

$$\text{Since } a = \frac{v_f - v_0}{t},$$

$$v_f = v_0 + at = 0 + (4 \text{ km/h/s})(15 \text{ s}) = 60 \text{ km/h}.$$

If you want to find the *average velocity*, v_{ave}, of the car over the 15 s time period, you can use the following formula.

$$v_{ave} = \frac{v_0 + v_f}{2} = \frac{0 + 60 \text{ km/h}}{2} = 30 \text{ km/h}$$

To find the *distance* traveled in the 15 s time period, you can use the formula

$$d = v_{ave} \cdot t.$$

To apply this formula correctly, the units must be compatible. Since the average velocity is given in kilometers per hour,

$$d = v_{ave} \cdot t = 30 \text{ km/h} \cdot \frac{15}{3600} \text{ h} = 0.125 \text{ km}.$$

Exercises

Solve. Approximate answers to the nearest hundredth.

1. A racing car starts from rest and accelerates at a constant rate along a straight road. After 5 s of constant acceleration, the car is traveling at a velocity of 85 km/h.
   a. What is the acceleration of the car over the 5 s time period?
   b. What is the average velocity of the car over the 5 s time period?
   c. Find the distance traveled by the car over the 5 s time period.
2. Starting from rest, a family car accelerates at a constant rate of 5 km/h/s along a straight road.
   a. At what velocity will it be traveling after 10 s of constant acceleration?
   b. Find the distance traveled by the car over the 10 s time period.
3. Sometimes it is important to know how far an automobile must travel to attain a certain velocity. The acceleration lanes that run roughly parallel to major highways allow cars to accelerate to highway speeds by the time they enter the highway. How long must an acceleration lane be to enable an automobile to accelerate at a constant rate from 36 km/h to 80 km/h in 10 s?