Chapter 1

Numbers and Variables

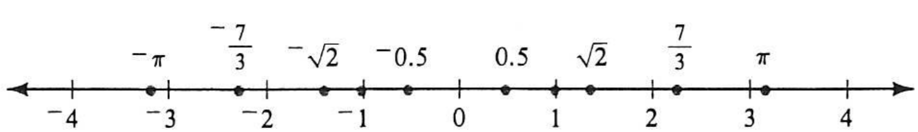
The Real Numbers

OBJECTIVES for Sections 1-1 through 1-3:
1. *To graph real numbers as points on a number line.*
2. *To show the order of real numbers.*
3. *To use set notation.*
4. *To specify and graph sets of real numbers.*

1–1 Number Lines

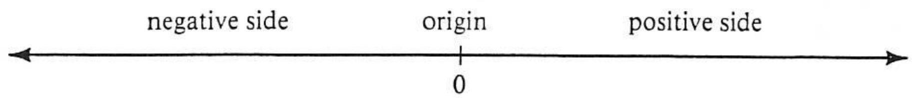
Whenever you count or measure, you use numbers. In order to picture numbers and the relationships among them, numbers can be placed in correspondence with points on a line. Such a picture is called a **number line.**

For example, in your study of mathematics thus far you have encountered *positive numbers* such as 1, 0.5, $\frac{7}{3}$, $\sqrt{2}$, and π. You also may have encountered *negative numbers* such as $^{-}1$ (read "negative one"), $^{-}0.5$, $^{-}\frac{7}{3}$, $^{-}\sqrt{2}$, and $^{-}\pi$. The figure below shows how these numbers can be pictured as points on a number line.

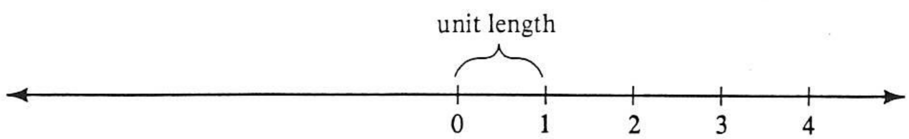


How do you construct a number line? The numbered steps on the following page outline a general procedure.

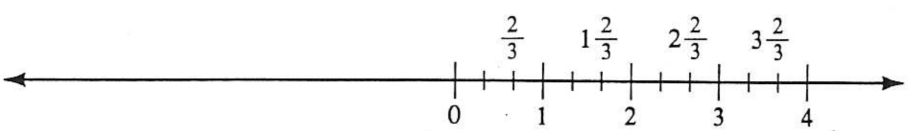
1. Choose a starting point on a line and label it 0. This point corresponds to the number zero and is called the **origin.** The origin separates a number line into two opposite sides, the *positive side* and the *negative side*. If the number line is horizontal, the side to the right of the origin is usually taken to be the positive side.



2. Choose any length to be one *unit*. Mark the point that is one unit to the positive side of 0 and label it 1. Using the same *unit length*, mark and label points to the positive side of 1 as 2, 3, and so on.

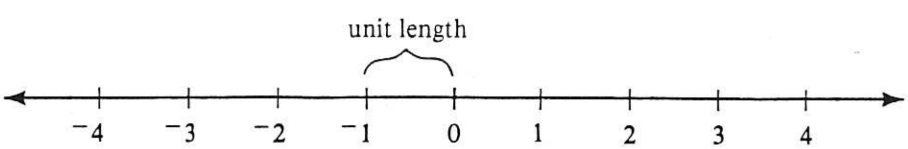


Using the points labeled 0, 1, 2, and so on as a reference, it is also possible to mark the points that correspond to fractions. For example, to label the points that correspond to $\frac{2}{3}$, $1\frac{2}{3}$, $2\frac{2}{3}$, and so on, separate each unit length into three segments of equal length. Then the point that is two thirds of the distance from 0 to 1 can be labeled $\frac{2}{3}$, the point that is two thirds of the distance from 1 to 2 can be labeled $1\frac{2}{3}$, and so on.

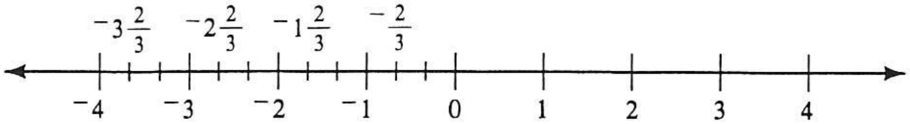


In general, a number that corresponds to a point on the positive side of a number line is called a **positive number.** A number that corresponds to a point on the negative side of a number line is called a **negative number.** To construct the negative side of a number line, continue the general procedure with the following step.

3. Using the same unit length as before, mark the point that is one unit to the negative side of the origin and label it $^{-}1$. Then mark and label points to the negative side of $^{-}1$ as $^{-}2$, $^{-}3$, and so on.



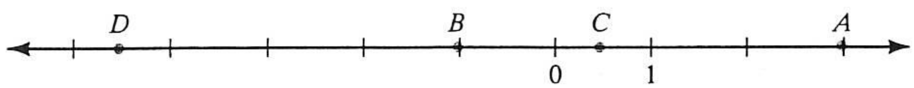
As on the positive side of the number line, the points labeled 0, $^{-}1$, $^{-}2$, and so on may be used as a reference in locating and labeling those points that correspond to negative fractions.



Note that the number zero itself is neither positive nor negative.

On a number line, the point that corresponds to a number is called the **graph of the number.** The number that corresponds to a point is called the **coordinate of the point.** A heavy dot is used to indicate the graph of a number.

EXAMPLE State the coordinate of each point graphed below.



SOLUTION A: 3; B: $^{-}1$; C: $\frac{1}{2}$; D: $^{-}4\frac{1}{2}$

Any number that is either a positive number, a negative number, or zero is called a **real number.** When you graph real numbers, you take the following for granted.

1. Each real number corresponds to exactly one point on a number line.
2. Each point on a number line corresponds to exactly one real number.

On a horizontal number line that is marked with positive numbers to the right, the real numbers increase from left to right and decrease from right to left. Thus a number line helps to determine the *order* of two real numbers. The *inequality symbols* $<$ and $>$ are often used when comparing numbers. For example:

$^{-}5$ is to the left of 1	1 is to the right of $^{-}5$
$^{-}5$ *is less than* 1	1 *is greater than* $^{-}5$
$^{-}5 < 1$	$1 > {}^{-}5$

To avoid confusing the symbols $<$ and $>$, notice that the greater number is placed at the greater, or open, end of the inequality symbol. The statements $^{-}5 < 1$ and $1 > {}^{-}5$ give the same information.

A number line also helps to determine if one real number is *between* two others. For example, on a number line you can see that 0 is between $^{-}5$ and 1 because $^{-}5$ is less than 0 *and* 0 is less than 1. You can use inequality symbols to write this relationship in the following ways.

$^{-}5 < 0$ and $0 < 1$	$1 > 0$ and $0 > {}^{-}5$
$^{-}5 < 0 < 1$	$1 > 0 > {}^{-}5$

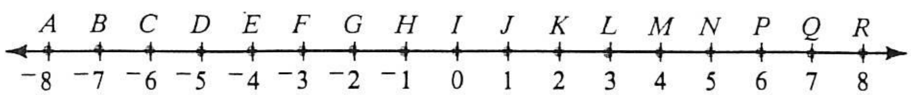
Because positive and negative numbers suggest opposite *directions*, they are sometimes called **directed numbers.** You use them for measurements that involve direction as well as size. For example:

a *gain* of \$10: 10	a *loss* of \$10: $^{-}10$
a temperature *rise* of 3°C: 3	a temperature *drop* of 3°C: $^{-}3$
12° *east* longitude: 12	12° *west* longitude: $^{-}12$

Sometimes to emphasize that a number like 10 is a positive number, it is called "positive ten" and is denoted by the symbol $^{+}10$. Note that the small signs $^{+}$ and $^{-}$ in the symbols $^{+}10$ and $^{-}10$ indicate the directions of the corresponding points from the origin on a number line. They do *not* indicate the operations of addition and subtraction.

Oral Exercises

Exercises 1–16 refer to the number line below.



Name the point that is the graph of the given number.

1. $^{-}5$ **2.** 1 **3.** 0 **4.** 7 **5.** 3 **6.** $^{-}3$

State the coordinate of the given point.

7. *Q* **8.** *G* **9.** *B* **10.** *P* **11.** *E* **12.** *I*

13. the point halfway between *D* and *F*
14. the point halfway between *G* and *K*
15. the point one third of the way from *I* to *P*
16. the point one fourth of the way from *C* to *K*

Translate each statement into words.

17. $^{-}6 < 1$ **18.** $^{-}2 > {}^{-}5$ **19.** $8 > 0 > {}^{-}10$ **20.** $^{-}9 < 4 < 7$

Written Exercises

Write a positive number for each measurement. Then write the opposite of that number and describe the measurement indicated by the opposite.

A
1. thirty wins
2. a gain of five yards
3. a profit of \$5000
4. a deposit of \$350
5. three floors up
6. five steps to the right
7. 400 m above sea level
8. ten seconds after liftoff
9. 90 km east
10. 35° north latitude

Graph the given numbers on a horizontal number line. Construct a separate number line for each exercise.

11. $^{-}5, {}^{-}2, 0, 2, 5$
12. $0, 1, 6, {}^{-}1, {}^{-}6$
13. $3, 4, 0, {}^{-}\frac{1}{2}, {}^{-}3$
14. $^{-}2, {}^{-}1.5, {}^{-}1, 2, 3$

15. $^{-}3, {}^{-}1.5, 0.5, 2, 3.5$

16. $1\frac{1}{2}, 2\frac{1}{2}, 0, \frac{^{-}1}{2}, {}^{-}3\frac{1}{2}$

17. $^{-}2\frac{1}{4}, \frac{^{-}3}{4}, 0, \frac{1}{4}, 3\frac{1}{4}$

18. $\frac{2}{3}, \frac{5}{3}, \frac{^{-}1}{3}, \frac{^{-}4}{3}, \frac{^{-}8}{3}$

Replace each __?__ with one of the symbols < or > to make a true statement.

19. $^{-}3$ __?__ 2

20. 1 __?__ $^{-}8$

21. $^{-}3$ __?__ $^{-}7$

22. $^{-}12$ __?__ $^{-}4$

23. $\frac{^{-}1}{5}$ __?__ $\frac{^{-}2}{5}$

24. $^{-}3$ __?__ $^{-}3.5$

25. $^{-}0.25$ __?__ $^{-}0.75$

26. $\frac{^{-}1}{2}$ __?__ $\frac{^{-}9}{10}$

27. $^{-}9$ __?__ 0 __?__ 9

28. $^{-}15$ __?__ $^{-}7$ __?__ 0

29. 11 __?__ 0 __?__ $^{-}2.5$

30. $^{-}4$ __?__ $^{-}4.5$ __?__ $^{-}4.75$

31. $\frac{^{-}7}{3}$ __?__ $\frac{^{-}5}{3}$ __?__ $\frac{^{-}2}{3}$

32. $\frac{^{-}5}{2}$ __?__ $^{-}3$ __?__ $\frac{^{-}7}{2}$

Write the given numbers in order from least to greatest.

B 33. $5, {}^{-}7, {}^{-}9, 4, 0, {}^{-}3$

34. $^{-}12, 1, {}^{-}1, {}^{-}16, 15, {}^{-}8$

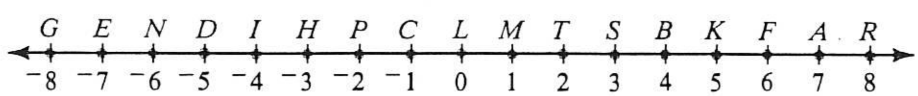
35. $\frac{^{-}1}{4}, \frac{^{-}1}{6}, \frac{^{-}1}{2}, \frac{^{-}1}{10}, \frac{^{-}1}{3}, \frac{^{-}1}{5}$

36. $\frac{^{-}2}{7}, \frac{3}{7}, \frac{^{-}5}{7}, \frac{^{-}9}{7}, \frac{4}{7}, \frac{^{-}1}{7}$

37. $4.5, {}^{-}1.5, {}^{-}2, 5.5, 0, {}^{-}2.5$

38. $^{-}1.25, {}^{-}1.5, {}^{-}1.15, {}^{-}1.05, {}^{-}1.75, {}^{-}1.1$

Exercises 39–54 refer to the number line below.



State the coordinate of the given point.

39. the point five eighths of the way from *L* to *R*
40. the point three eighths of the way from *G* to *L*
41. the point one half of the way from *M* to *T*
42. the point one half of the way from *N* to *D*
43. the point one fourth of the way from *P* to *B*
44. the point three fourths of the way from *C* to *K*
45. the point one third of the way from *B* to *R*
46. the point one third of the way from *N* to *C*
47. the point three fifths of the way from *T* to *F*
48. the point two fifths of the way from *G* to *P*
49. the point two thirds of the way from *D* to *S*
50. the point five eighths of the way from *N* to *B*

C 51. the point between *L* and *F* that is twice as far from *L* as it is from *F*
52. the point between *G* and *B* that is three times as far from *G* as it is from *B*
53. the point to the left of *P* that is twice as far from *C* as it is from *P*
54. the point to the right of *M* that is half as far from *M* as it is from *I*

1–2 Sets and Symbols

A **set** is a collection of objects. The objects are called the **members,** or **elements,** of the set. You use braces, { }, to indicate that a set is being named. Within the braces, you separate the members of the set by commas. For example, to indicate "the set whose members are 1, 3, 5, and 7," you write

$$\{1, 3, 5, 7\}.$$

To indicate that 5 is a member of this set, you use the symbol $\in$, *is a member of*, and write

$$5 \in \{1, 3, 5, 7\}.$$

If you wish to indicate that 9 is not a member of this set, you use the symbol $\notin$, *is not a member of*, and write

$$9 \notin \{1, 3, 5, 7\}.$$

Sets that contain exactly the same members are called **equal sets.** You use the symbol =, *is equal to,* to indicate that sets are equal. The order in which you list the members of a set does not matter, and so

$$\{1, 3, 5, 7\} = \{3, 1, 7, 5\}.$$

On the other hand, the sets {1, 3, 5, 7} and {1, 3, 5, 9} do *not* contain exactly the same members. Therefore, you use the symbol $\neq$, *does not equal*, and write

$$\{1, 3, 5, 7\} \neq \{1, 3, 5, 9\}.$$

Although the sets {1, 3, 5, 7} and {1, 3, 5, 9} are not equal, there is an important relationship between them. The figure that follows shows a pairing that assigns to each member of each set *one and only one* member of the other set.

$$\begin{array}{c} \{1,\ 3,\ 5,\ 7\} \\ \updownarrow\ \updownarrow\ \updownarrow\ \updownarrow \\ \{1,\ 3,\ 5,\ 9\} \end{array}$$

Such a pairing of the members of two sets is called a **one-to-one correspondence.** A one-to-one correspondence of great importance in mathematics is that between the set of points on a line and the set of real numbers. (Recall Section 1–1.)

When you list all the members of a set, you *specify* the set by **roster.** You can also specify a set by writing within the braces a **rule,** or description, that identifies the members of the set. For example,

$$\{1, 3, 5, 7\} = \{\text{the odd numbers between 0 and 8}\}.$$

A third way to specify a set of *numbers* is to graph the numbers on a number line. The set of points corresponding to a set of numbers is called the **graph of the set.** When graphing sets of numbers, recall that, on a horizontal number line that is marked with positive numbers to the right, "greater than" means "to the right of," while "less than" means "to the left of."

Often you speak of a set of numbers that are "between" two given numbers. In such cases, note that the given numbers indicate the *boundaries* of the set, but they are not themselves *members* of the set.

EXAMPLE Graph each set of numbers.

a. {1, 3, 5, 7}

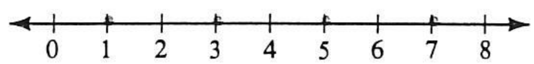
b. {the odd numbers between 0 and 8}

c. {the real numbers between 0 and 8}

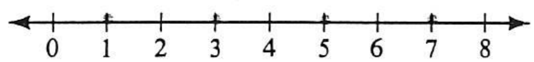
d. {the real numbers greater than 0}

e. {the real numbers less than or equal to 8}

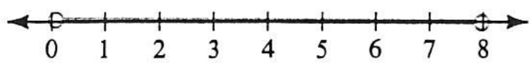
SOLUTION **a.**



b.



c.



d.

e.

Notice in parts (c), (d), and (e) of the Example that heavy shading is used to show that all points on the indicated portion of the number line belong to the graph. Open dots and portions of the line not shaded show points that do *not* belong to the graph. A heavy arrowhead shows that the graph continues without end in the indicated direction.

Capital letters are often used to name sets. Thus you can write

$S = \{1, 3, 5, 7\}$ and $T = \{\text{the real numbers between 0 and 8}\}$.

If every member of a set S is also a member of a set T, then S is called a **subset** of T. You use the symbol $\subset$, *is a subset of*, to indicate that one set is a subset of another. Thus for the sets S and T just specified,

$$S \subset T.$$

Every set is a subset of itself.

$$S \subset S \qquad T \subset T$$

To indicate that one set is *not* a subset of another set, you use the symbol $\not\subset$, *is not a subset of*.

$$T \not\subset S$$

The set that contains no members is called the **empty set,** or the **null set.** You denote the empty set by the symbol $\emptyset$. As an example,

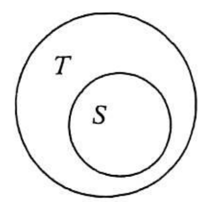
$$\emptyset = \{\text{the negative numbers between 0 and 8}\}.$$

The empty set is a subset of every set.

$$\emptyset \subset S \qquad \emptyset \subset T \qquad \emptyset \subset \emptyset$$

Be careful not to confuse the empty set $\emptyset$ with $\{0\}$. The set $\{0\}$ contains exactly one member, namely 0.

Often it is useful to draw a diagram that shows how certain sets are related. One such type of diagram is the **Venn diagram.** (Venn diagrams are named after John Venn, an English mathematician who was among the first to use them extensively.) For example, to show that a set S is a subset of a set T, you can use a Venn diagram such as the one at the right.

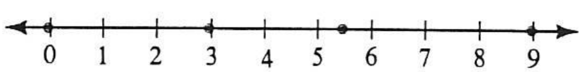


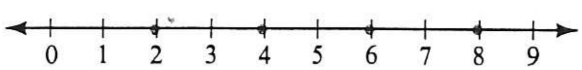
$S \subset T$

Oral Exercises

Specify each set by roster.

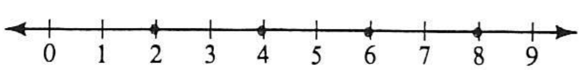
1. {the days of the week}
2. {the last five letters of the English alphabet}
3. {the months that have thirty-one days}
4. {the months that have thirty-two days}
5. {the positive numbers between $^{-}10$ and 0}
6. {the even numbers between 1 and 15}

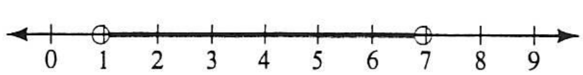
7. 

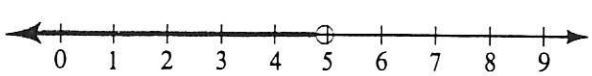
8. 

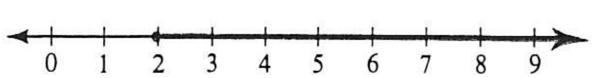
Specify each set by rule.

9. {summer, fall, winter, spring}
10. {January, June, July}
11. {a, b, c, d, e, f}
12. {a, e, i, o, u}
13. {12, 14, 16, 18}
14. {11, 13, 15, 17, 19}

15. 

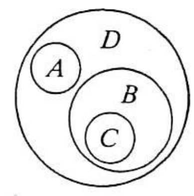
16. 

17. 

18. 

Tell whether each statement is true or false. Exercises 27–30 refer to the Venn diagram at the right.

19. $\{1, 2, 3, 4, 5\} = \{3, 1, 5, 2, 4\}$
20. $\{1, 2, 3, 4, 5\} =$ {the real numbers between 0 and 6}
21. $3 \in \{1, 2, 3, 4, 5\}$
22. $6 \notin \{1, 2, 3, 4, 5\}$
23. $\{1, 2\} \subset \{1, 2, 3, 4, 5\}$
24. $\{5, 6\} \subset \{1, 2, 3, 4, 5\}$
25. $\{1, 2, 3, 4, 5\} \not\subset \emptyset$
26. $\emptyset \subset \{1, 2, 3, 4, 5\}$
27. $A \subset D$
28. $B \subset C$
29. $A \not\subset B$
30. $C \subset D$



Exs. 27–30

Written Exercises

Replace each _?_ with one of the symbols $\in$ or $\subset$ to make a true statement.

A

1. 2 _?_ $\{0, 2, 4, 6\}$
2. $\{2, 4\}$ _?_ $\{0, 2, 4, 6\}$
3. $\{0\}$ _?_ $\{0, 2, 4, 6\}$
4. $\{4, 0, 6, 2\}$ _?_ $\{0, 2, 4, 6\}$
5. 0 _?_ $\{0\}$
6. $\emptyset$ _?_ $\{0\}$
7. $\emptyset$ _?_ $\emptyset$
8. $\emptyset$ _?_ $\{0, 2, 4, 6\}$

Graph each set of numbers.

9. $\{^-5, ^-3, 0, 3\}$
10. $\{^-6, ^-4, ^-2, 1, 3, 5\}$
11. $\left\{^-3\frac{1}{2}, ^-\frac{1}{2}, 2\frac{1}{2}, 4\frac{1}{2}\right\}$
12. $\{^-4.5, ^-0.5, 0, 1.5, 2.5\}$
13. {the odd numbers between 4 and 14}
14. {the even numbers between 1 and 11}
15. {the multiples of 3 between $^-10$ and 0}
16. {the multiples of 4 between $^-10$ and 1)}
17. {the real numbers between $^-10$ and 10}
18. {the real numbers between $^-4$ and 5}
19. {the real numbers greater than $^-2$}
20. {the real numbers less than or equal to $3\frac{1}{2}$}
21. {the real numbers greater than $^-2$ and less than or equal to 5}
22. {the real numbers greater than or equal to $^-5\frac{1}{2}$ and less than $^-1$}
23. {the positive real numbers}
24. {the negative real numbers}
25. {the positive real numbers less than 6}
26. {the negative real numbers greater than $^-1$}
27. {the positive and negative real numbers}
28. {the real numbers that are neither positive nor negative}

Draw a Venn diagram to illustrate each statement.

29. $A \subset B$

30. $B \subset A$

B 31. $A \subset B$ and $B \subset A$

32. $A \not\subset B$ and $B \not\subset A$

33. $A \subset B$ and $B \subset C$

34. $A \subset C$ and $B \subset C$, but $A \not\subset B$ and $B \not\subset A$

For Exercises 35–39, let $S = \{a, b, c\}$.

35. List all the subsets of S that contain exactly one member.
36. List all the subsets of S that contain exactly two members.
37. List all the subsets of S that contain exactly three members.
38. List all the subsets of S that contain no members.
39. What is the total number of subsets of S?

C 40. Let $T = \{a, b, c, d\}$. List all the subsets of T. What is the total number of subsets of T?

41. Let $U = \{a, b, c, d, e\}$. List all the subsets of U. What is the total number of subsets of U?

42. If a set V contains six members, what is the total number of subsets of V?

43. If a set W contains eight members, what is the total number of subsets of W?

Marjorie Lee Browne

1914–1979

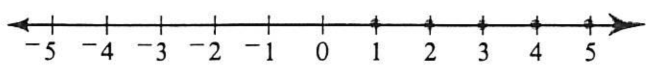
1–3 Sets of Numbers

The set of numbers that is used in counting is called the set of **natural numbers,** or **counting numbers.** Often this set is referred to as *N*. Since it would be impossible to make a roster of *all* the natural numbers, you can specify the set *N* as follows.

$$N = \{1, 2, 3, 4, 5, \ldots\}$$

The three dots after the 5 are read "and so on" and indicate that the pattern established by the listed numbers continues without end.

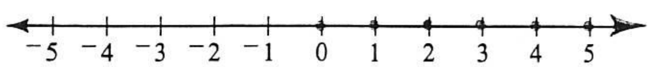
The set *N* may also be specified by the following graph.



Here the heavy arrowhead indicates that the graph continues indefinitely to the right following the pattern established by the numbers that are graphed with heavy dots.

When you expand the set of natural numbers to include the number 0, you obtain the set *W* of **whole numbers.**

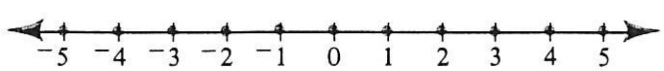
$$W = \{0, 1, 2, 3, 4, 5, \ldots\}$$



The set *J*, where

$$J = \{\ldots, {}^{-}3, {}^{-}2, {}^{-}1, 0, 1, 2, 3, \ldots\},$$

is called the set of **integers.** Here the dots at the beginning and end of the list indicate that the pattern of the numbers continues without end in *both* directions. Therefore, to graph the set of integers you need to use a heavy arrowhead at both ends of the number line.



EXAMPLE Specify each set by roster.

a. {the natural numbers greater than 9}
b. {the odd integers between $^{-}4$ and 4}
c. {the whole numbers less than 15}

SOLUTION **a.** $\{10, 11, 12, 13, 14, \ldots\}$
b. $\{{}^{-}3, {}^{-}1, 1, 3\}$
c. $\{0, 1, 2, 3, \ldots, 14\}$

In part (c) of the Example, the three dots are read "and so on through" and indicate that the pattern established by the first four listed numbers continues until you reach the last listed number.

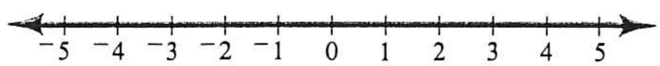
A **rational number** is any number that can be expressed as the quotient of two integers, provided that the divisor is not 0. Thus, the following numbers are rational numbers.

$$0 = \frac{0}{1} \qquad 7 = \frac{7}{1} \qquad \frac{2}{9} \qquad 3\frac{1}{2} = \frac{7}{2} \qquad 0.21 = \frac{21}{100}$$

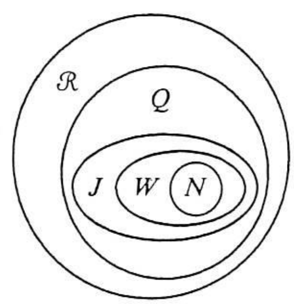
The set of rational numbers is often referred to as Q.

As you might imagine, it would be impossible to label all the rational numbers on a number line. Even if you *could* do so, there would still be many points of the number line that would not be labeled. These points correspond to numbers such as $\sqrt{2}$ and π. Such numbers *cannot* be expressed as the quotient of two integers, and they are called **irrational numbers.**

Together the rational numbers and the irrational numbers make up the set of *real numbers*, which is denoted by the symbol $\mathcal{R}$. Recall that a real number was previously defined as any number that is either a positive number, a negative number, or zero. Thus you can graph the set of real numbers by graphing the entire number line, as follows.



To illustrate the relationships among the sets N, W, J, Q, and $\mathcal{R}$, you can use a Venn diagram such as the one at the right.



Each of the sets N, W, J, Q, $\mathcal{R}$, and the set of irrational numbers is called an **infinite set** because the process of counting its members would never come to an end. A set that is not an infinite set is called a **finite set.**

Oral Exercises

Match.

1. {1, 2, 3, 4, . . .}
2. {. . . , 1, 2, 3, 4}
3. {1, 2, 3, 4}
4. {$^-4$, $^-3$, $^-2$, $^-1$}
5. {. . . , $^-4$, $^-3$, $^-2$, $^-1$}
6. {$^-4$, $^-3$, $^-2$, $^-1$, . . .}
7. {$^-4$, $^-3$, $^-2$, $^-1$, . . . , 4}
8. {$^-4$, $^-2$, 0, 2, 4, . . .}
9. {. . . $^-3$, $^-1$, 1, 3}
10. {0, 2, 4}
11. {1, 3}
12. Ø

a. {the negative integers less than 5}
b. {the natural numbers less than 5}
c. {the even integers greater than $^-5$}
d. {the integers between $^-5$ and 5}
e. {the odd integers less than 5}
f. {the natural numbers greater than $^-5$}
g. {the even whole numbers less than 5}
h. {the odd natural numbers less than 5}
i. {the negative integers greater than $^-5$}
j. {the positive integers less than $^-5$}
k. {the integers less than 5}
l. {the integers greater than $^-5$}

List all the sets N, W, J, Q and $\mathcal{R}$ of which each number is a member. Each number may be a member of more than one set.

13. 0
14. 7
15. $^-3$
16. $\frac{5}{6}$
17. $\frac{^-8}{4}$
18. $^-\sqrt{5}$

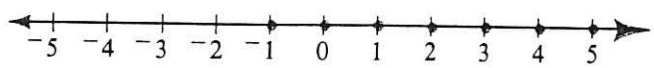
Tell whether each set is finite or infinite.

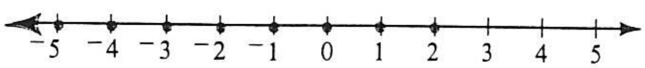
19. {the whole numbers greater than 8}
20. {the natural numbers less than 8}
21. {the integers between $^-9$ and 9}
22. {the integers between 0 and 1}
23. {the rational numbers between 0 and 1}
24. {the negative irrational numbers}

Written Exercises

Specify each set by roster.

A

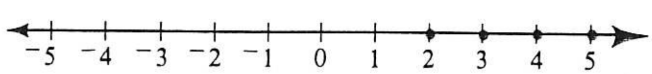
1. 

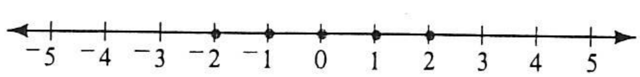
2. 

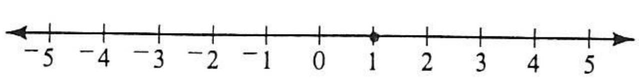
3. {the natural numbers less than 7}
4. {the whole numbers greater than 24}
5. {the integers greater than $^-10$ and less than 2}
6. {the positive integers between $^-10$ and 2}
7. {the negative integers greater than or equal to $^-2$}
8. {the even integers less than 3}
9. {the natural numbers between 9 and 100}
10. {the odd integers greater than $^-50$ and less than 50}

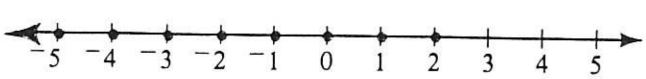
Specify each set by rule.

11. {1, 2, 3, 4, 5}
12. {$^-3$, $^-2$, $^-1$, 0, 1, 2}
13. {$^-6$, $^-5$, $^-4$, $^-3$, $^-2$, . . .}
14. {. . . , $^-9$, $^-7$, $^-5$, $^-3$, $^-1$}
15. {2, 4, 6, 8, . . . , 100}
16. {$^-50$, $^-49$, $^-48$, $^-47$, . . . , 50}
17. {. . . , $^-6$, $^-3$, 0, 3, 6, . . .}
18. {. . . , $^-10$, $^-5$, 0, 5, 10, . . .}

19. 

20. 

21. 

22. 

Graph each set of numbers.

23. $\{\ldots, {}^-2, {}^-1, 0, 1, 2, 3\}$
24. $\{\ldots, {}^-4, {}^-2, 0, 2, 4, \ldots\}$
25. {the natural numbers less than 10}
26. {the negative integers greater than $^-7$}
27. {the integers less than or equal to 1}
28. {the whole numbers greater than 4}
29. {the even integers between $^-7$ and 7}
30. {the natural numbers greater than $^-4$ and less than 4}

Replace each $\underline{\ ?\ }$ with one of the words *All*, *Some*, or *No* to make a true statement that has the widest application.

EXAMPLE $\underline{\ ?\ }$ integers are real numbers.

SOLUTION Replacing the $\underline{\ ?\ }$ with either *All* or *Some* yields a true statement. However, the statement with *All* has the widest application. Thus: *All* integers are real numbers.

B 31. $\underline{\ ?\ }$ rational numbers are real numbers.
32. $\underline{\ ?\ }$ real numbers are irrational numbers.
33. $\underline{\ ?\ }$ irrational numbers are integers.
34. $\underline{\ ?\ }$ natural numbers are integers.
35. $\underline{\ ?\ }$ real numbers are whole numbers.
36. $\underline{\ ?\ }$ rational numbers are integers.
37. $\underline{\ ?\ }$ integers are rational numbers.
38. $\underline{\ ?\ }$ rational numbers and irrational numbers are real numbers.
39. $\underline{\ ?\ }$ real numbers are rational numbers or irrational numbers.
40. $\underline{\ ?\ }$ whole numbers are natural numbers.
41. $\underline{\ ?\ }$ natural numbers are irrational numbers.
42. $\underline{\ ?\ }$ rational numbers are negative integers.

For Exercises 43–52, $A = \{{}^-3, {}^-2, {}^-\frac{1}{2}, 0, 1, 3\}$ and $B = \{{}^-2, \frac{1}{2}, 1, \frac{3}{2}, 2, \pi\}$. Specify each of the following sets by roster.

43. {the integers that are members of A but not of B}
44. {the integers that are members of B but not of A}
45. {the integers that are members of either A or B}
46. {the integers that are members of both A and B}
47. {the rational numbers that are members of either A or B}
48. {the rational numbers that are members of both A and B}

C **49.** {the nonintegers that are members of both A and B}

50. {the nonintegers that are members of B but not of A}

51. {the positive integers that are members of neither A nor B}

52. {the positive integers that are not members of both A and B}

Self-Test 1

VOCABULARY

number line (p. 1)
origin (p. 2)
positive number (p. 2)
negative number (p. 2)
graph of a number (p. 3)
coordinate of a point (p. 3)
real number (p. 3)
directed number (p. 3)
set (p. 6)
member or element of a set (p. 6)
equal sets (p. 6)
one-to-one correspondence (p. 6)
specify a set by roster (p. 6)
specify a set by rule (p. 6)
graph of a set of numbers (p. 6)
subset (p. 7)
empty set or null set (p. 8)
Venn diagram (p. 8)
natural number or counting number (p. 11)
whole number (p. 11)
integer (p. 11)
rational number (p. 12)
irrational number (p. 12)
infinite set (p. 12)
finite set (p. 12)

Graph the given numbers on a horizontal number line. Construct a separate number line for each exercise.

1. $^-4, ^-1, 0, 3, 5$ **2.** $^-4.5, ^-2, ^-0.5, \frac{1}{2}, 2.5$ *Obj. 1, p. 1*

Replace each _?_ with one of the symbols < or > to make a true statement.

3. 2 _?_ $^-9$ **4.** $^-7.5$ _?_ 3 **5.** $\frac{^-2}{5}$ _?_ $\frac{^-3}{5}$ _?_ $\frac{^-4}{5}$ *Obj. 2, p. 1*

Tell whether each statement is true or false.

6. $0 \in \emptyset$ **7.** $\{2, 1\} \subset \{1\}$ **8.** $\{0, 1\} = \{1, 0\}$ *Obj. 3, p. 1*

Graph each set of numbers.

9. {the real numbers between $^-4$ and 1} *Obj. 4, p. 1*

10. {the positive integers greater than $^-3$ and less than 3}

Check your answers with those at the back of the book.

Careers

Oceanography

Oceanographers study the characteristics of the ocean in order to predict its behavior and to safely utilize it as a source of food, chemicals, and energy. Since there are so many aspects of the ocean to be studied, many oceanographers choose to specialize in a single field.

Physical oceanographers study tide and wave patterns and the ability of ocean waters to conduct sound, light, and heat waves. *Biological oceanographers* are concerned with the living organisms of the ocean and with the relationship of these organisms to their environment. *Geological oceanographers* concentrate on the formation and physical characteristics of the ocean floor, while *chemical oceanographers* study the composition of the water and sediment.

While some oceanographers are college professors or work in industry, most are involved in research. This research may take place in laboratories or on boats and platforms in the ocean, where easy underwater exploration can take place. Research trips to the ocean may vary in length from a few days to several months, depending on the amount and type of data to be gathered.

EXAMPLE An oceanographer needs to know the depth of a ship that is lying on the ocean floor. *Sonar*, a system of sending out sound waves and monitoring their return, is one method of determining the depth of an object in the ocean. A sound wave is sent down to the ship and returns in 0.076 s. Given that the speed of sound in water is 1490 m/s, what is the depth of the ship?

SOLUTION The total time needed for the sound wave to travel to the sunken ship *and back* is 0.076 s. Thus, the time needed for the sound wave to reach the ship is *half* the total time, or 0.038 s. The depth can be found by multiplying this time by the speed of sound in water, which is given as 1490 m/s.

$$\begin{aligned}\text{Depth} &= \underbrace{\text{Time to reach ship}} \times \underbrace{\text{Speed of sound}}\\ &= 0.038\ \text{s} \times 1490\ \text{m/s}\\ &= 56.62\ \text{m}\end{aligned}$$

Therefore, the depth of the ship is *approximately* 57 m.

Numerical and Variable Expressions

OBJECTIVES for Sections 1-4 through 1-6:
1. To simplify numerical expressions and evaluate variable expressions.
2. To simplify and evaluate expressions that contain grouping symbols.
3. To simplify and evaluate expressions that contain exponents.

1–4 Simplifying and Evaluating Expressions

One kind of expression that is used in algebra is called a *numerical expression*. A **numerical expression,** or **numeral,** is simply a name for a number. The number is called the **value of the expression.** For example, since the numerical expressions 4×9 and 36 name the same number, thirty-six, they have the same value. To show that they have the same value, you use the equals sign, =. You write

$$4 \times 9 = 36,$$

which is read "four times nine *equals* (or *is equal to*) thirty-six." To show that two numerical expressions do *not* have the same value, you use the symbol $\neq$. For example, you can write

$$4 \times 9 \neq 37,$$

which is read "four times nine *is not equal to* (or *does not equal*) thirty-seven."

Note that a raised dot may also be used as a multiplication symbol.

$$4 \times 9 \text{ may be written } 4 \cdot 9.$$

When you replace a numerical expression with the simplest, or most common, name of its value, you **simplify the expression.** Thus, since 36 is the most common name for the number thirty-six, you simplify the numerical expression 4×9, or $4 \cdot 9$, when you replace it with 36.

In simplifying a numerical expression, you use the following basic principle.

> ***Substitution Principle***
>
> Changing the numeral by which a number is named in an expression does not change the value of the expression.

EXAMPLE 1 Simplify.

a. $7 + 19 + 140$ **b.** $4 \cdot 17 + 23$

SOLUTION

a. $\underbrace{7 + 19}_{} + 140$
$\underbrace{26 + 140}_{}$
166

b. $\underbrace{4 \cdot 17}_{} + 23$
$\underbrace{68 + 23}_{}$
91

Another kind of symbol used in algebra is a *variable*. A variable is a symbol that is used to represent one or more numbers. The set of numbers that a variable may represent is called the domain, or replacement set, of the variable. Each number in the domain is called a value of the variable. A variable may be a letter, such as n, or a different type of symbol.

An expression that contains a variable is called a variable expression. A variable expression may contain more than one variable, and it may also contain other symbols, including numerals. When you write a product that contains a variable, you usually omit the multiplication symbol.

$3 \times n$ is usually written $3n$.
$y \times z$ is usually written yz.

When you replace each variable in a variable expression with one of its values and simplify the resulting numerical expression, you evaluate the expression, or find the value of the expression.

EXAMPLE 2 Evaluate each expression when $a = 24$.

a. $a - 5$ **b.** $3a$ **c.** $2a + 9$ **d.** $40 - \frac{a}{3}$

SOLUTION

a. $a - 5 = 24 - 5 = 19$
b. $3a = 3 \times 24 = 72$
c. $2a + 9 = 2 \times 24 + 9 = 48 + 9 = 57$
d. $40 - \frac{a}{3} = 40 - \frac{24}{3} = 40 - 8 = 32$

EXAMPLE 3 Find the greatest value of the expression $k + 7$ if $k \in \{4, 10, 15\}$.

SOLUTION

When $k = 4$,
$k + 7 = 4 + 7$
$= 11$

When $k = 10$,
$k + 7 = 10 + 7$
$= 17$

When $k = 15$,
$k + 7 = 15 + 7$
$= 22$

Therefore, the greatest value of $k + 7$ over the given domain is 22.

Often a numerical expression or a variable expression is referred to as a mathematical expression.

Oral Exercises

Tell whether each statement is true or false.

1. $6 \times 9 = 9 \times 6$
2. $7 \times 0 = 0 \times 5$
3. $8 + 0 \neq 8 - 0$
4. $2 \div 1 \neq 1 \div 2$
5. $6 \times 3 = 6 \div 3$
6. $10 \times 1 = 10 \div 1$
7. $\frac{1}{3} \times 12 = 12 \div 3$
8. $8 \div 2 \neq \frac{8}{2}$
9. $3 \times 5 \neq 5 + 5 + 5$
10. $25 - 9 = 2 \times 2 \times 2 \times 2$

Simplify.

11. $25 + 92$
12. $53 - 12$
13. $32 \cdot 3$
14. $\frac{144}{12}$

Evaluate each expression when $x = 6$.

15. $x + 18$
16. $9 + x$
17. $x - 5$
18. $30 - x$
19. $12x$
20. $\frac{1}{3}x$
21. $\frac{x}{2}$
22. $\frac{24}{x}$

Find the greatest value of each expression if $y \in \{2, 4, 6, 8, 10\}$.

23. $y + 2$
24. $y - 2$
25. $4y$
26. $\frac{1}{2}y$
27. $16 - y$
28. $2y + 8$
29. $\frac{y - 2}{2}$
30. $\frac{2}{y + 2}$

Written Exercises

Simplify.

A

1. $4392 + 227 + 46$
2. $1990 - 1776$
3. $3 \times 6 \times 50$
4. $\frac{415}{5}$
5. $12 - 0.49$
6. $9.5 + 4 + 8.32$
7. $6.5 \div 0.13$
8. 1.2×0.08
9. $52\frac{3}{4} + 97\frac{1}{2}$
10. $24 - \frac{7}{8}$
11. $2\frac{2}{3} \times 3\frac{1}{2}$
12. $62 \div \frac{1}{2}$

Evaluate each expression when $a = 4$, $b = 8$, and $c = 18$.

13. $a + 19$
14. $20 - c$
15. $10b$
16. $\frac{b}{5}$
17. $2b + 5$
18. $13 - \frac{c}{3}$
19. $\frac{3}{c - 3}$
20. $\frac{b + 2}{2}$

Evaluate each expression when $a = 4$, $b = 8$, and $c = 18$.

21. $a + c$
22. $c - b$
23. bc
24. $4ab$
25. $3a + b$
26. $3b + a$
27. $\frac{6}{a + b}$
28. $\frac{a + b}{6}$
29. $a + c - b$
30. $c - b - a$
31. abc
32. $ab + c$
33. $\frac{bc}{a}$
34. $\frac{ac}{b}$
35. $c - \frac{b}{a}$
36. $b - \frac{c}{a}$

Evaluate each expression when the variables have the given values.

B

37. $a + b;\ a = \frac{1}{2},\ b = \frac{1}{3}$
38. $a - b;\ a = 1\frac{1}{2},\ b = 1\frac{1}{4}$
39. $c - d + 2;\ c = 0.24,\ d = 0.08$
40. $c - d + 1;\ c = 1.04,\ d = 0.6$
41. $fg + 1;\ f = \frac{3}{4},\ g = \frac{1}{2}$
42. $fg - 1;\ f = 1\frac{2}{3},\ g = \frac{3}{4}$
43. $\frac{m}{n};\ m = \frac{3}{4},\ n = \frac{1}{2}$
44. $\frac{m}{n};\ m = 1\frac{1}{3},\ n = 1\frac{1}{6}$
45. $2 + \frac{p}{q};\ p = 0.75,\ q = 0.05$
46. $2 - \frac{p}{q};\ p = 0.6,\ q = 1.5$
47. $\frac{1}{t};\ t = \frac{2}{3}$
48. $\frac{1}{t};\ t = 1\frac{2}{3}$
49. $\frac{1}{u} + \frac{1}{v};\ u = \frac{1}{2},\ v = \frac{1}{3}$
50. $\frac{1}{u} - \frac{1}{v};\ u = 2\frac{1}{3},\ v = 3\frac{1}{2}$
51. $xy;\ x = \frac{1}{3},\ y = \frac{3}{4}$
52. $5xy;\ x = 1\frac{1}{5},\ y = 3$

53. Write four variable expressions that contain the variable z and that have a value of 1 when $z = 3$.
54. How many variable expressions contain the variable w and have a value of 0 when $w = 3$?

Find values of a and b that make each statement true.

C

55. The expressions $a + b$ and $a - b$ have the same value.
56. The expressions ab and $\frac{a}{b}$ have the same value.
57. The expression $a - b$ has the same value as b.
58. The expression $\frac{a}{b}$ has the same value as b.

59. What is true of the value of $\frac{1}{n}$ and of the value of $n + \frac{1}{n}$ as the value of n increases, if $n \in \{1, 2, 3, 4, 5, \ldots\}$?
60. What is true of the value of $\frac{1}{n}$ and of the value of $n + \frac{1}{n}$ as the value of n decreases, if $n \in \{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\}$.

1–5 Grouping Symbols

Simplifying or evaluating an expression frequently involves more than one operation. In such cases parentheses, ( ), are often used to indicate which operations are to be performed first. Different groupings may produce expressions for different numbers. For example,

$$(3 \cdot 7) + 9 = 21 + 9 = 30,$$

while

$$3 \cdot (7 + 9) = 3 \cdot 16 = 48.$$

A **grouping symbol** is a device, such as a pair of parentheses, that is used to enclose an expression. Brackets, [ ], and braces, { }, may also be used as grouping symbols.

A product such as $3 \cdot (7 + 9)$ is usually written without the multiplication symbol simply as $3(7 + 9)$. You read this expression formally as "three times *the quantity* seven plus nine." Similarly, a product such as $3 \cdot 16$ may be written in one of the following ways.

$$3(16) \qquad \text{or} \qquad (3)16 \qquad \text{or} \qquad (3)(16)$$

Variables may be grouped in the same way that numerals are grouped. When you group a product that involves a variable, you usually omit the grouping symbols.

$(5n) + 2$ is usually written $5n + 2$.
$16 \div (4y)$ is usually written $16 \div 4y$.
$(8a) - (5b)$ is usually written $8a - 5b$.

A fraction bar is both a division symbol and a grouping symbol. When operation symbols appear above or below the fraction bar, those operations are to be performed before the division. For example,

$$\frac{6 + 18}{2 + 6} = \frac{24}{8} = 3,$$

and,

$$\text{when } q = 21, \frac{q + 7}{q - 7} = \frac{21 + 7}{21 - 7} = \frac{28}{14} = 2.$$

EXAMPLE 1 Evaluate each expression when $x = 3$.

a. $8(x - 2) - 5$ **b.** $54 \div 2x$ **c.** $\frac{x + 9}{4x}$

SOLUTION

a. $8(x - 2) - 5 = 8(3 - 2) - 5 = 8(1) - 5 = 8 - 5 = 3$

b. $54 \div 2x = 54 \div 2(3) = 54 \div 6 = 9$

c. $\frac{x + 9}{4x} = \frac{3 + 9}{4(3)} = \frac{12}{12} = 1$

If an expression contains more than one grouping symbol, first simplify the numeral in the innermost grouping symbol. Then work toward the outermost grouping symbol until you have simplified the entire expression.

EXAMPLE 2 Simplify $9[24 - 3(5 + 1)]$.

SOLUTION
$$\begin{aligned} 9[24 - 3(5 + 1)] &= 9[24 - 3(6)] \\ &= 9[24 - 18] \\ &= 9(6) \\ &= 54 \end{aligned}$$

Oral Exercises

Tell whether each statement is true or false.

1. $2(3 + 4) \neq 2(3) + 4$
2. $(2 + 2)2 = 2 + 2(2)$
3. $\frac{10 + 4}{2} = \frac{10}{2} + 4$
4. $\frac{12 + 4}{4 + 2} \neq \frac{12}{4} + \frac{4}{2}$
5. $12 \div (1 + 1) \neq (12 \div 1) + 1$
6. $4 - (2 \div 1) = 4 \div (2 - 1)$
7. $8 + (4 + 1) = (8 + 4) + 1$
8. $(14 - 6) - 5 = 14 - (6 - 5)$
9. $\frac{24}{6 \div 2} \neq \frac{24}{6} \cdot 2$
10. $\frac{10 \cdot 3}{2} = \frac{10}{2} \cdot 3$
11. $36 \div (6 \div 2) = (36 \div 6) \div 2$
12. $(7 \cdot 5) \cdot 2 \neq 7 \cdot (5 \cdot 2)$

Simplify.

13. $5(3) + 4$
14. $47 - 2(5)$
15. $7(8 - 2)$
16. $(3 \cdot 3) + (4 \cdot 4)$
17. $\frac{3 \cdot 8}{6 + 2}$
18. $3(7) + \frac{15}{3}$
19. $\frac{3(9) + 1}{10 + 4}$
20. $(8 - 2)(3 + 5)$

Written Exercises

Simplify.

A

1. $21(5) + 4$
2. $85 - 3(9)$
3. $23(4) - 6(13)$
4. $17(3) + 10(3)$
5. $16(25 - 14)$
6. $(26 + 9)7$
7. $\frac{90 - 10}{30 - 10}$
8. $\frac{80 - 62}{15 + 57}$
9. $2(19) + \frac{48}{3}$
10. $\frac{144}{3} - 14(3)$
11. $\frac{3(9) - 7}{7(5) + 5}$
12. $\frac{9(9) + 12(12)}{4(4) + 3(3)}$
13. $[24 - 9(2)]2$
14. $8[3(5) - 4]$
15. $12[2(2) + 4(4)]$
16. $[6(14) + 2(13)]9$
17. $[5(13) - 11] \div 3$
18. $[6(10) + 7(0)] \div 4$

Evaluate each expression when $v = 0$, $w = 1$, $x = 2$, $y = 3$, and $z = 5$.

19. $3x + 5y$ 20. $7z - 3y$ 21. $2(v + 3) - 2$ 22. $16 - 3(z - 5)$

23. $xy - z$ 24. $vw + yz$ 25. $x(z - v) - y$ 26. $(w + y)v + yz$

27. $\frac{xz - 1}{x + 1}$ 28. $\frac{y + z}{x} + w$ 29. $xy - \frac{w}{z}$ 30. $xyz - \frac{v}{w}$

Simplify.

B 31. $[7(3) - 6] + [5 + 3(4)]$

32. $2[2(5) - 7] + [9 + 5(5)]$

33. $6[9(3) - 17][6(6) - 5(7)]$

34. $[6(9) + 7][6 + 9(7)]0$

35. $8\{[5(6) - 7(3)] - 9\}$

36. $\{5 + 3[2(5) - 4] - 3\}2$

37. $\{[2(25) + 5(70)] \div [80(15) \div 6(10)]\} + [25(12 \div 6)]$

38. $\{[(15 - 3)(7 + 5)] + 6(6)\} - 9\{(15 \div 3)[5(4) \div (12 - 2)]\}$

Replace each _?_ with one of the symbols = or ≠ to make a true statement.

39. $3[(5 + 1)(6 \div 2)]$ _?_ $[18 + (144 \div 8)] + 3$

40. $[3(5 - 2)]4$ _?_ $96 \div [32 \div (5 - 3)]$

41. $\frac{[9 + 3(13)] \div 12}{4 - (6 \div 2)} + 1$ _?_ $27 - \frac{40 + 4}{8 - 6}$

42. $\frac{[5(10) - 2] - [3(14) - 6(6)]}{(36 \div 2) + 2}$ _?_ $\frac{30 - 3}{1 + 2} + 3$

43. $\frac{4(3) + 22}{8 + (27 \div 3)} - \frac{8 - 4}{2 + 2}$ _?_ $\frac{7(6) + 6}{30 - 2(3)} - 1$

44. $\frac{\{[3(5) - 7] + 2(2)\} + 40}{\{[2(16) - 5] - 3(3)\} - 5}$ _?_ $\frac{(54 \div 3) + 2(3)}{(60 \div 5) - 2(3)}$

45. $2\left\{\frac{[5 - 2(2)] + 8(3)}{[(24 \div 6) + 4] - 3}\right\}$ _?_ $[(2 + 5) - (30 \div 6)]\left(\frac{12 - 2}{5 - 3}\right)$

46. $\left\{\frac{90 - [13(18) \div 3]}{[7 + 6(4)] - 19}\right\}4$ _?_ $\{[18(3) \div 9] - 5\}\left(\frac{24 \div 2}{2(3)}\right)$

Let $m \in \{1, 2, 3, \ldots, 10\}$. Find the greatest and least values of each expression.

C 47. $4m - 3$ 48. $60 - 5m$ 49. $m(2m - 1)$ 50. $\frac{12}{m + 1}$

Let $m \in \{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{10}\}$. Find the greatest and least values of each expression.

51. $2m + 3$ 52. $10 - 3m$ 53. $m(1 - m)$ 54. $\frac{2m + 1}{m}$

Computer Exercises

The optional Computer Exercises that appear throughout this book can be worked using BASIC or any other programming language. Students using BASIC may find Appendix B at the back of the book summarizing the fundamental concepts and terminology of BASIC useful.

Write a program to evaluate each expression when $n = 6$.

1. $2n + 2$
2. $2(n + 2)$
3. $\frac{n}{2}$
4. $\frac{n + 2}{2}$
5. $\frac{2}{n - 2}$
6. $\frac{n + 2}{n - 2}$
7. $\frac{2n + 2}{2n - 2}$
8. $\frac{2n + 2}{2n} - 2$
9. Let $n \in \{1, 2, 3, 4, 5, \ldots, 100\}$. Write a program to evaluate the expression $\frac{2n - 1}{2n + 1}$ over this domain. What is true of the value of this expression as the value of n increases?

1–6 Exponents and Order of Operations

When one factor of a product is used a number of times, you may use an *exponent* to simplify the notation. For example:

$$\underbrace{3 \times 3 \times 3 \times 3 \times 3}_{5 \text{ factors}} = 3^5$$

You read the expression 3^5 as "three to the fifth power" or as "the fifth power of three." In this expression, 3 is called the **base** and 5 is called the **exponent.** The number 3^5, or 243, is called a **power** of 3. The expression 3^5 is called the **exponential form** of the power.

Other powers of 3 can be defined and written as follows.

$3^1 = 3$	"three to the first power"
$3^2 = 3 \times 3 = 9$	"three to the second power" or "three squared" or "the square of three"
$3^3 = 3 \times 3 \times 3 = 27$	"three to the third power" or "three cubed" or "the cube of three"
$3^4 = 3 \times 3 \times 3 \times 3 = 81$	"three to the fourth power"

Notice the special language that is associated with the second and third powers of a number. In fact, the expression 3^2 is *usually* read as "three squared" or as "the square of three" because 3^2 can represent the area of a square with sides of length 3 units. Similarly, the expression 3^3 is usually read as "three cubed" or as "the cube of three" because 3^3 can represent the volume of a cube with edges of length 3 units.

EXAMPLE 1 Simplify.

a. 2^4 b. $3^2 \times 4^3$ c. 2×5^3 d. $(2 \times 5)^3$

SOLUTION

a. $2^4 = 2 \times 2 \times 2 \times 2 = 16$

b. $3^2 \times 4^3 = (3 \times 3) \times (4 \times 4 \times 4) = 9 \times 64 = 576$

c. $2 \times 5^3 = 2 \times (5 \times 5 \times 5) = 2 \times 125 = 250$

d. $(2 \times 5)^3 = (10)^3 = 10 \times 10 \times 10 = 1000$

When simplifying expressions, these steps should be followed in order.

Order of Operations

1. First simplify expressions within grouping symbols.
2. Then simplify powers.
3. Then simplify products and quotients in order from left to right.
4. Then simplify sums and differences in order from left to right.

EXAMPLE 2 Simplify $29 + 32 \div (8 - 6)^2$.

SOLUTION

$$\begin{aligned} 29 + 32 \div (8 - 6)^2 &= 29 + 32 \div (2)^2 \\ &= 29 + 32 \div 4 \\ &= 29 + 8 \\ &= 37 \end{aligned}$$

Exponents may also be used in variable expressions. For example:

$$\underbrace{n \times n \times n \times n}_{4 \text{ factors}} = n^4$$

EXAMPLE 3 Evaluate the expression $100 - a(a + 1)^3$ when $a = 2$.

SOLUTION

$$\begin{aligned} 100 - a(a + 1)^3 &= 100 - 2(2 + 1)^3 \\ &= 100 - 2(3)^3 \\ &= 100 - 2(27) \\ &= 100 - 54 \\ &= 46 \end{aligned}$$

In general, if x denotes any real number and n denotes any positive integer, the expression x^n is defined as follows.

$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}$$

Oral Exercises

State the exponential form of each expression.

1. $7 \times 7 \times 7 \times 7 \times 7$
2. $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
3. $2 \times 2 \times 3 \times 3 \times 3 \times 3$
4. $3 \times 3 \times 11 \times 11$
5. $x \cdot x$
6. $y \cdot y \cdot y$
7. $x \cdot x \cdot y \cdot y \cdot y$
8. $x \cdot x \cdot x \cdot y \cdot y$
9. $5 \cdot a \cdot a$
10. $3 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot c$
11. $(n + 1)(n + 1)$
12. $11(m - n)(m - n)(m - n)$

Simplify.

13. 5^2
14. 2^3
15. $(0.5)^2$
16. $(0.1)^2$
17. 2×3^2
18. $2^2 \times 3$
19. $(2 \times 3)^2$
20. $2^2 \times 3^2$

Evaluate each expression when $a = 3$ and $b = 2$.

21. a^2
22. $2a$
23. ab^2
24. $(ab)^2$
25. $a + b^2$
26. $(a + b)^2$
27. $a^3 - b^3$
28. $(a - b)^3$

Written Exercises

Write each expression in exponential form.

A

1. $x \cdot x \cdot x \cdot x \cdot x$
2. $y \cdot y \cdot y \cdot y \cdot y \cdot y$
3. $7 \cdot a \cdot a \cdot a \cdot a$
4. $2 \cdot b \cdot b \cdot b \cdot b \cdot c$
5. $(x - y)(x - y)(x - y)$
6. $5 \cdot r \cdot r \cdot r \cdot (s + t)(s + t)$
7. m squared
8. the cube of n
9. p plus the square of q
10. the cube of the quantity r plus s
11. one half the fifth power of t
12. the fifth power of one half t

Simplify.

13. 3^5
14. 2^6
15. 7^4
16. 11^3
17. $27 + 2^4$
18. $(18 + 5)^2$
19. $2^2 + 3^3 + 4^4$
20. $5^4 - 9^2 - 2^5$
21. $12^2 - 2(5)^2$
22. $4(3^2) + 6^3$
23. $(12^2 - 5^3)2^3$
24. $5(5^3 - 4^3)$
25. $\frac{6^2 - 4^2}{6 - 2^2}$
26. $\frac{7^2 - 5^2}{2^4 + 3^2}$
27. $\frac{(3 + 7)^3}{7^2 - 3^2}$
28. $\frac{10^2 - 5^2}{(10 - 5)^2}$

Evaluate each expression when the variable has the given value.

29. $2y^2 - 3y;\ y = 5$
30. $z^2 - z - 1;\ z = 3$
31. $10 - 5a + 3a^2;\ a = 1$
32. $b^2 - 8b + 9;\ b = 0$
33. $\frac{c^2 - 2c - 35}{c - 5};\ c = 7$
34. $\frac{d^2 + d - 20}{d^2 + 2d - 15};\ d = 5$

Simplify.

B

35. $2^6 \div 2^2 \div 2^3 \div 2$
36. $3^4 - 3^2 \div 3^2 - 3$
37. $4(9 + 2) - 24 \div 2(4)$
38. $(8 - 1)12 \div 3(4) - 6$
39. $75 + 36 \div 3(5 - 3)^2$
40. $84 \div 7 + 5(12 - 2^2)$
41. $3(5^2 - 1) - 10(3^2) \div 5$
42. $(5^3 - 11^2)10 \div 5(2)^2 + 3$
43. $(12 - 3)^2 + (7^2 - 2)^2 - (13 - 2)^2$
44. $[(8^2 - 5^2) - (7^2 - 6^2) - (3^2 + 4^2)]^2$
45. $[(0.4)^2 \div (0.2)^2] - (0.4 \div 0.2)^2$
46. $[(1.1)^2 - (0.7)^2] \div [(0.2)(0.3)]^2$
47. $\left(\frac{1}{2}\right)^2 + \left(\frac{5 - 3}{5}\right)^2 + \left(\frac{3 + 4}{10}\right)^2$
48. $\left(\frac{2^3}{2^3 + 1}\right)^2 \div \left(\frac{3^2 + 1}{3^2}\right)^2$

Evaluate each expression when the variable has the given value.

49. $4a - 5a^2;\ a = 0.3$
50. $b^2 - 9(b - 1)^2;\ b = 1.1$
51. $12x - 9(1 - x)^2;\ x = \frac{2}{3}$
52. $y^2 + y + 2;\ y = \frac{3}{5}$
53. $0.5(1 - 2m + 2m^2);\ m = 0.5$
54. $(n + 1)^2(n^2 + 2n + 1);\ n = 0.1$

Replace each _?_ with one of the symbols +, −, ×, or ÷ to make a true statement.

C

55. 12 _?_ 2 _?_ 2 _?_ 8 _?_ 4 = 18
56. 36 _?_ 3 _?_ 12 _?_ 6 _?_ 1 = 36
57. 64 _?_ $\frac{1}{2}$ _?_ 8 _?_ 4 _?_ 4 _?_ $\frac{1}{2}$ = 8
58. 18 _?_ 6 _?_ $\frac{1}{3}$ _?_ 2 _?_ 6 _?_ $\frac{1}{3}$ = 4
59. 24 _?_ 6 _?_ 8 _?_ 0.5 _?_ 16 _?_ 0.5 = 12
60. 12 _?_ 0.25 _?_ 16 _?_ 0.5 _?_ 0.5 _?_ 24 = 2

Self-Test 2

VOCABULARY

numerical expression or numeral (p. 17)
value of a numerical expression (p. 17)
simplify a numerical expression (p. 17)
substitution principle (p. 17)
variable (p. 18)
domain or replacement set of a variable (p. 18)
value of a variable (p. 18)
variable expression (p. 18)
evaluate or find the value of a variable expression (p. 18)
mathematical expression (p. 18)
grouping symbol (p. 21)
base (p. 24)
exponent (p. 24)
power (p. 24)
exponential form of a power (p. 24)

Find the least value of each expression if $x \in \{1, 2, 3, 4\}$.

1. $6x + 5$ 2. $15 - 2x$ 3. $\frac{60}{x + 1}$ *Obj. 1, p. 17*

Evaluate each expression when $a = 2$ and $b = 6$.

4. $a + 7(b - 1)$ 5. $8b \div 3a$ 6. $\frac{ab + 4}{ab - 4}$ *Obj. 2, p. 17*

Simplify.

7. 5^4 8. $10^2 - 3(2)^2$ 9. $8 \div 2 + 2(8 - 2^3)$ *Obj. 3, p. 17*

Check your answers with those at the back of the book.

ON THE CALCULATOR

Does your calculator follow the correct order of operations in simplifying expressions? Experiment with your calculator by entering the following example exactly as it appears here:

$$7 + 3 \times 5 =$$

If your calculator displays the answer 22, it evaluated the expression correctly by following the order of operations outlined on page 25. Your calculator has an algebraic operating system.

If your calculator displays the answer 50, it performed the operations in the order in which they were entered. This type of calculator can compute accurately, but you must enter the numbers and operations in the proper sequence. To obtain a correct answer on this type of calculator, you must enter the preceding example as follows:

$$3 \times 5 + 7 =$$

Use a calculator to simplify each expression.

1. $26 + 87 \div 3$
2. $48 - 35 + 17$
3. $95 + 17 \times 4$
4. $9 \times 27 + 12 + 35$
5. $8(46 - 19)$
6. $(67 + 77) \div 12$
7. $172 - 6 \times 19$
8. $\frac{532}{19 \times 4}$
9. $278 - 14^2$
10. $16 + 23^2 - 1$
11. $4 + 8 \times 17^2$
12. $2^3(24^2 - 7)$

READING ALGEBRA Numerical Expressions

Algebra is the primary language of mathematics. As you study algebra, you will learn to read and write expressions in this language.

As you may know, there is often more than one way to translate a given mathematical expression into words. The following are some familiar examples of *numerical expressions*. After each expression you see several different ways that it can be translated into a word phrase.

$9 + 2$

"nine plus two"
"the sum of nine and two"
"nine increased by two"
"two more than nine"

$7 - 4$

"seven minus four"
"the difference when four is subtracted from seven"
"seven decreased by four"
"four less than seven"

12×3

"twelve multiplied by three"
"the product of twelve and three"
"twelve times three"

$10 \div 2$

"ten divided by two"
"the quotient when ten is divided by two"

$\frac{1}{3} \times 4$

"one third times four"
"one third of four"
"the product of one third and four"

$\frac{4}{3}$

"four thirds"
"four divided by three"
"the quotient when four is divided by three"

Exercises

Write two different word phrases than can represent each numerical expression.

1. $14 - 8$
2. $\frac{16}{7}$
3. 4×15
4. $18 + 9$

Translate each word phrase into a numerical expression.

5. six times twenty
6. the sum of fourteen and three
7. one tenth of ninety
8. nine decreased by six
9. four more than twelve
10. the product of sixteen and seven
11. nine thirds
12. zero plus six

Mathematical Sentences and Problem Solving

OBJECTIVES for Sections 1-7 through 1-9:

1. *To solve an open sentence in one variable over a specified domain and to graph its solution set.*
2. *To translate numerical relationships stated in word phrases into mathematical expressions.*
3. *To translate numerical relationships stated in word sentences into mathematical sentences.*

1–7 Equations and Inequalities

A statement that indicates a relationship between two mathematical expressions is called a **mathematical sentence.** If a mathematical sentence states that two expressions name the same number, then the sentence is an **equation,** and the equals sign, =, is used to show the relationship. The two expressions are called the **sides of the equation.** For example,

$$4 \cdot 7 = 28$$

is an equation; $4 \cdot 7$ is the *left* side of the equation, and 28 is the *right* side.

A mathematical sentence that is formed by placing an *inequality symbol* between two mathematical expressions is called an **inequality.** Among the commonly used inequality symbols are $<$, $>$, $\leq$, $\geq$, and $\neq$. The following are examples of inequalities that make use of these symbols.

$3 < 8$	read	"3 *is less than* 8"
$9 > 4$	read	"9 *is greater than* 4"
$6 + 7 \leq 18$	read	"6 plus 7 *is less than or equal to* 18"
$32 - 9 \geq 23$	read	"32 minus 9 *is greater than or equal to* 23"
$48 \div 16 \neq 4$	read	"48 divided by sixteen *is not equal to* 4"

As with equations, the expressions to the left and to the right of an inequality symbol are called the **sides of the inequality.**

If both sides of a mathematical sentence are *numerical* expressions, then the sentence may be classified as either *true* or *false*. For example, the mathematical sentences

$$5 + 9 = 14 \qquad 7 < 9 \qquad 6 \neq 40 \div 8$$

are true, whereas the mathematical sentences

$$5 + 9 = 13 \qquad 7 > 9 \qquad 5 \neq 40 \div 8$$

are false. Note that the mathematical sentences

$$12 \geq 7 \qquad \text{and} \qquad 12 \geq 12$$

are *both* true. A "greater than or equal to" sentence such as these is true *either* if the first number is greater than the second *or* if the first number is equal to the second. Similarly, a "less than or equal to" sentence is true either if the first number is *less than* the second or if the first number is equal to the second.

A mathematical sentence that contains at least one variable is called an **open mathematical sentence,** or simply an **open sentence.** The following are examples of open sentences.

$$2x - 5 = 35 \qquad a > 11 \qquad 24 \neq n \div 3$$

Generally a domain is specified for each variable in an open sentence. Any value of the variables for which the open sentence is true is called a **solution,** or **root,** of the open sentence *over the given domain*. The set of *all* such values is called the **solution set** of the open sentence, and each member of the solution set is said to **satisfy** the open sentence. When you determine the solution set of an open sentence over a given domain, you **solve** the open sentence over that domain.

EXAMPLE 1 Solve $x - 5 = 6$ if $x \in \{10, 11, 12, 13, 14\}$.

SOLUTION Replace x with each of its values in turn. Determine whether the resulting sentence is true or false.

$x - 5 = 6$	
$10 - 5 = 6$	False
$11 - 5 = 6$	True
$12 - 5 = 6$	False
$13 - 5 = 6$	False
$14 - 5 = 6$	False

Therefore, the solution set is $\{11\}$.

Sometimes the given domain for a variable in an open sentence is an infinite set. Although in such cases it is impossible to replace the variable with *each* of its values, it still may be possible to determine the solution set of the open sentence.

EXAMPLE 2 Solve $3n = 7$ if $n \in$ {the positive integers}.

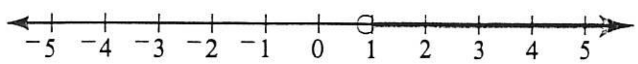
SOLUTION Since the possible values of n are the positive integers, or 1, 2, 3, 4, . . . , the corresponding values of $3n$ are the positive multiples of 3, or 3, 6, 9, 12, ... Since 7 is not one of these numbers, the equation $3n = 7$ has no solution over the domain of the positive integers.

Therefore, the solution set is $\emptyset$.

The **graph of an open sentence** is the graph of its solution set.

EXAMPLE 3 Solve $x + 8 > 9$ over $\mathcal{R}$ and graph its solution set.

SOLUTION In order for the sentence $x + 8 > 9$ to be true, x must represent a real number that is greater than 1.
Therefore, the solution set is $\{x: x > 1\}$.
The graph of the solution set is the following.



The colon within the brackets in Example 3 is read as "such that." You read the expression

$$\{x: x > 1\}$$

as, "the set of all x such that x is greater than 1." This notation is often called **set-builder notation.**

Oral Exercises

Solve each open sentence if $x \in \{0, 1, 2, 3, 4, 5\}$.

1. $x - 3 = 2$ **2.** $9 - x = 9$ **3.** $3x = 12$ **4.** $\frac{1}{2}x = 8$

5. $x + 1 > 5$ **6.** $10 - x < 7$ **7.** $2x > 12$ **8.** $5x < 20$

9. $x + 2 = 2x$ **10.** $4x > x + 8$ **11.** $6x < 0$ **12.** $x = 2x - 1$

Solve each open sentence if $z \in$ {the positive integers}.

13. $z + 3 = 5$ **14.** $15 - z = 9$ **15.** $4z = 28$ **16.** $\frac{1}{3}z = 5$

17. $z + 1 < 6$ **18.** $5 + z > 8$ **19.** $3z \geq 18$ **20.** $\frac{1}{2}z \leq 5$

21. $2z = z$ **22.** $z - 1 < z$ **23.** $^{-}3 < z < 3$ **24.** $5 \leq z < 9$

Written Exercises

Solve each open sentence if $n \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$.

A **1.** $2 + n = 6$ **2.** $12 = n + 9$ **3.** $5 = n - 2$ **4.** $10 - n = 7$

5. $6n = 12$ **6.** $24 = 3n$ **7.** $3 = \frac{1}{3}n$ **8.** $\frac{n}{2} = 4$

9. $2n + 3 = 11$ **10.** $18 - 2n = 16$ **11.** $2n + 3 < 11$ **12.** $18 - 2n > 16$

13. $n^2 = n^3$ **14.** $n^2 > n$ **15.** $2n < n^2$ **16.** $n^2 \neq 4$

Solve each open sentence if $y \in$ {the positive real numbers} and graph the solution set.

17. $y \geq 2$
18. $y \leq 5$
19. $0.5 < y$
20. $3\frac{1}{2} > y$
21. $1 < y < 6$
22. $7 \geq y \geq 2$
23. $6\frac{1}{2} > y > 3$
24. $1.5 < y < 5$
25. $y + 3 = 9$
26. $4 + y > 5$
27. $7.5 - y = 2$
28. $y - 1 = 2\frac{1}{2}$
29. $4y + 1 = 17$
30. $4y - 1 \geq 11$
31. $y(y + 3) = 4$
32. $y(y + 2) > 15$

B

33. $3 \leq y + 1 \leq 7$
34. $5 \geq y - 3 \geq 1$
35. $8 > y - 2 > 4$
36. $1 < 1 + y < 6$
37. $y + 2 \geq y$
38. $y \geq y + 1$
39. $y + 2 \leq y + 1$
40. $y + 1 > y - 1$

Let $a \in \{1, 2\}$ and $b \in \{3, 4\}$. In each open sentence, replace a and b with all possible pairs of values from their respective domains. Tell whether the resulting statements are true or false.

EXAMPLE $2a + 3b = 14$

SOLUTION

$2(1) + 3(3) = 14$	False	$2(2) + 3(3) = 14$	False
$2(1) + 3(4) = 14$	True	$2(2) + 3(4) = 14$	False

41. $a + b = 6$
42. $b - a = 2$
43. $7 - b = 2a$
44. $3a = 2b - 5$
45. $b < 3a$
46. $a + 6 \geq 2b$
47. $3a + b > 7$
48. $8 \leq 3b - a$

Write two different open sentences for which the solution set over $\mathcal{R}$ is the given set.

C

49. $\{5\}$
50. $\{\frac{1}{3}\}$
51. $\mathcal{R}$
52. $\emptyset$
53. {all real numbers except 0}
54. {all real numbers except 1}

ON THE CALCULATOR

You can use your calculator to evaluate variable expressions for given values of the variables if you keep in mind the order in which the values must be entered, as discussed on page 28.

Use a calculator to evaluate each expression when $a = 4.5$, $b = 0.9$, and $c = 1.2$.

1. $a + bc$
2. $a(b + c)$
3. $(a + b)c$
4. $a + b^2$
5. $c^2 - b$
6. $(8a)(2c)$
7. $2b + 5c$
8. $a + 3(a - b)$
9. $a + 3b - c - a$
10. $5a - 3b - 6b - 2a$
11. $2(a - b + c) - 3(c - b)$
12. $3(a^2 + b) + 4(a^2 + b)$

PROGRAMMING IN BASIC

You can use a computer to solve open sentences over a given finite domain by programming the computer to test each value in the domain to see if it satisfies the open sentence. For example, the program that follows will provide a graph of the open sentence $X + 8 > 9$ over the domain $\{0, 1, 2, \ldots, 15\}$.

```
10  PRINT "TO GRAPH AN OPEN SENTENCE"
20  PRINT "IN ONE VARIABLE:"
30  PRINT "(SENTENCE IS IN LINE 70.)"
40  PRINT
50  REM *DOMAIN
60  FOR X = 0 TO 15
70  IF X + 8 > 9 THEN 120
80  REM *ELSE
90  PRINT "-";
100  GOTO 130
110  REM *THEN
120  PRINT "*";
130  NEXT X
140  PRINT ">"
150  PRINT "0"
160  END
```

Notice that the program illustrates several of the fundamental structures of the BASIC computer programming language:

FOR-NEXT loop
IF-THEN conditional transfer
GOTO unconditional transfer

The program also utilizes the PRINT and REM (remark) statements. If you need additional information about any of these items, see Appendix B at the back of the book.

Exercises

1. Type in and RUN the program as given. Compare the result with the graph shown on page 32.

2. What is the purpose of lines 140 and 150?

Change line 70 as indicated, then RUN the revised program.

3. 70 IF X − 5 = 6 THEN 120

4. 70 IF 3 * X = 7 THEN 120

5. `70 IF 3 * X = 12 THEN 120`

6. `70 IF X + 3 = 5 THEN 120`

7. `70 IF 15 - X = 9 THEN 120`

8. `70 IF 3 * X >= 18 THEN 120`

9. `70 IF X + 1 < 6 THEN 120`

10. `70 IF 5 <= X THEN 120`

11. `70 IF 4 * X > X + 8 THEN 120`

12. `70 IF X > 2 * X - 1 THEN 120`

13. `70 IF 3 <= X + 1 AND X + 1 <= 7 THEN 120`

14. `70 IF 5 >= X - 3 AND X - 3 >= 1 THEN 120`

If possible, save this program for later use.

1–8 Words into Symbols: Expressions

Often you must translate a *word phrase* about numbers into a numerical or variable expression. In order to do so, you must be able to translate each part of the word phrase into an appropriate mathematical symbol. For example, the word phrase

forty-six *decreased by* nineteen

can be translated into the numerical expression

$$46 - 19.$$

Sometimes a single mathematical expression might be used to translate many different word phrases. For example, the variable expression

$$n + 50$$

might represent any of the following word phrases:

the *sum* of a number n and fifty
a number n *increased by* fifty
fifty *more than* a number n
the *total* of a number n and fifty
a number n *plus* fifty

EXAMPLE 1 Write a mathematical expression for each word phrase.

a. the difference when four is subtracted from seventy-two
b. the product of fifteen and a number y
c. seventeen times the quantity ninety-four plus twelve
d. two less than the quotient when a number m is divided by 9

SOLUTION **a.** $72 - 4$ **b.** $15y$ **c.** $17(94 + 12)$ **d.** $\frac{m}{9} - 2$

Mathematical expressions often represent real-life situations.

EXAMPLE 2 Ann is a years old now. Write a variable expression for each word phrase.

a. Ann's age five years from now

b. four years less than half Ann's present age

c. twice Ann's age three years ago

d. David's age, if the sum of Ann's age and David's age is thirty

SOLUTION a. $a + 5$ b. $\frac{1}{2}a - 4$ c. $2(a - 3)$ d. $30 - a$

EXAMPLE 3 Write a variable expression for each word phrase.

a. the number of months in y years

b. the total value in cents of d dimes and q quarters

SOLUTION a. One year consists of $12 \cdot 1$ months, or 12 months.
Two years consist of $12 \cdot 2$ months, or 24 months.
Therefore, y years consist of $12 \cdot y$ months, or $12y$ months.

b. The value of one dime is 10 cents.
The value of d dimes is $10 \cdot d$ cents, or $10d$ cents.
The value of one quarter is 25 cents.
The value of q quarters is $25 \cdot q$ cents, or $25q$ cents.
The *total* value of d dimes and q quarters is the *sum* of the values of the dimes and quarters, or $10d + 25q$ cents.

Oral Exercises

Match.

1. twelve less than a number n
2. twelve decreased by a number n
3. a number n increased by twelve
4. a number n multiplied by twelve
5. one twelfth of a number n
6. twelve more than half a number n
7. the difference when twice a number n is subtracted from twelve
8. twice the difference when twelve is subtracted from a number n
9. the product of twelve and the sum of a number n and twelve
10. the quotient when the total of a number n and twelve is divided by twelve
11. twelve divided by the sum of twelve and a number n
12. the sum of a number n and twelve, divided by the product of n and twelve

a. $12n$

b. $2(n - 12)$

c. $12 - n$

d. $\frac{n + 12}{12}$

e. $12(n + 12)$

f. $\frac{1}{12}n$

g. $12 - 2n$

h. $\frac{n + 12}{12n}$

i. $n - 12$

j. $\frac{12}{12 + n}$

k. $n + 12$

l. $\frac{1}{2}n + 12$

Replace each _?_ with a variable expression to make a true statement.

13. The value in cents of n nickels is _?_.
14. The value in cents of x dollars is _?_.
15. The number of seconds in m minutes is _?_.
16. The number of hours in d days is _?_.
17. If x is a positive integer, the next greater positive integer is _?_.
18. If y is a positive odd integer, the next greater positive odd integer is _?_.
19. If you were a years old six years ago, right now you are _?_ years old.
20. If you will be z years old four years from now, right now you are _?_ years old.

Written Exercises

Write a variable expression for each word phrase.

A

1. five more than twice a number x
2. eight less than the square of a number y
3. forty-five decreased by one fourth a number z
4. the cube of a number w, increased by seventy-nine
5. the product of seventeen and the square of the difference when seven is subtracted from a number m
6. the quotient when the fourth power of the sum of a number n and sixteen is divided by five
7. the sum of a number p and eleven, multiplied by the difference when fourteen is subtracted from a number q
8. the quotient when three times a number r is divided by the product of two and a number s
9. the total of a number a and twice a number b, divided by the total of a number c and the fifth power of a number d
10. the product when the sum of a number g and the square of a number h is multiplied by the difference when a number k is subtracted from the cube of a number j
11. the number of days in w weeks
12. the number of minutes in h hours
13. the number of days in h hours
14. the number of minutes in s seconds
15. the total value in cents of n nickels and d dimes
16. the total value in cents of p pennies, d dimes, and q quarters
17. the total value in dollars of n nickels and q quarters
18. the total value in dollars of p pennies, d dimes, and x dollars

Write a variable expression for each word phrase.

19. the sum of a positive integer j and the next greater positive integer
20. the sum of a positive even integer k and the next greater positive even integer
21. the product of a positive odd integer x and the next greater positive odd integer
22. the quotient when a positive integer y is divided by the next greater positive integer
23. your age in years, if your sister is a years old and the sum of your age and your sister's age is twenty-four years
24. your mother's age in years, if you are b years old and your mother is two years older than three times your age
25. twice your age six years ago, if you are m years old now
26. one fourth your age twenty years from now, if you are n years old now

Represent the answer to each question in terms of the given variable(s).

B 27. The sum of two numbers is forty-eight. One number is x. What is the other number?

28. The product of two numbers is thirty-six. One number is y. What is the other number?
29. The difference between two numbers is ten. If the lesser number is a, what is the greater number?
30. The quotient when one number is divided by another is four. If the lesser number is z, what is the greater number?
31. Linda has six more quarters than nickels. If she has n nickels, what is the total value in cents of her quarters and nickels?
32. Darryl has three fewer dimes than quarters. If he has d dimes, what is the total value in cents of his dimes and quarters?
33. Ken has nine times as many pennies as dimes. If he has d dimes, what is the total value in cents of his pennies and dimes?
34. Alice has six times as many dollar bills as quarters. If she has q quarters, what is the total value in dollars of her dollar bills and quarters?
35. Sam is m years old. Lisa is three times as old as Sam was five years ago. How old is Lisa?
36. Claudia is n years old. Joel is one third as old as Claudia will be in ten years. How old is Joel?
37. Two sides of an isosceles triangle each measure y centimeters. The third side is five centimeters longer. What is the perimeter of the triangle?
38. The width of a rectangle is three centimeters shorter than the length. If the width is x centimeters, what is the area of the rectangle?

C 39. A passenger train travels at a speed of v km/h. A freight train travels at a speed that is x km/h slower. How much farther can the passenger train travel than the freight train in h hours?

40. Today Jan drove her truck for h hours at a speed of r km/h. Yesterday she drove x fewer hours at a speed that was y km/h faster. What is the total distance that Jan drove yesterday and today?

41. Green peppers cost n cents per pound. This is z cents per pound less than the cost of red peppers. What is the total cost of g pounds of green peppers and r pounds of red peppers?

42. Last week Carl bought m pounds of potatoes for his restaurant at a cost of y cents per pound. This week he bought n more pounds than last week at a cost that was z cents per pound greater. What is the difference between the amount he paid for the potatoes last week and the amount he paid this week?

Computer Exercises For students with computer experience

1. Write a program that will compute the equivalent number of seconds when you input any number of minutes. The output should be in the form: x minutes $= y$ seconds.
2. Write a program that will compute the equivalent number of hours and minutes when you input any number of minutes. The output should be in the form: x minutes $= y$ hours z minutes.
3. Write a program that will compute the total value of a collection of coins when you input the number of pennies, nickels, dimes, quarters, half dollars, and dollars in the collection. The output should display the total value in dollars using the correct \$ and . notation.

ON THE CALCULATOR

Solve each equation if $n \in \{1, 2, 3, 4\}$.

1. $\frac{n(n+1)}{2} = 1$

2. $\frac{n(n+1)}{2} = 1 + 2$

3. $\frac{n(n+1)}{2} = 1 + 2 + 3$

4. $\frac{n(n+1)}{2} = 1 + 2 + 3 + 4$

Let $n \in$ {the natural numbers}. Guess what value of n satisfies each equation, then use a calculator to check your guess.

5. $\frac{n(n+1)}{2} = 1 + 2 + 3 + \ldots + 10$

6. $\frac{n(n+1)}{2} = 1 + 2 + 3 + \ldots + 24$

1–9 Words into Symbols: Sentences

Just as you can translate a word phrase into a mathematical expression, you also can translate a *word sentence* into a mathematical sentence.

EXAMPLE 1 Write a mathematical sentence for each word sentence.

a. The sum of seven and twice a number n is twenty-five.

b. The quotient of a number y divided by three is greater than four times y, decreased by eleven.

SOLUTION **a.** $7 + 2n = 25$

b. $\frac{y}{3} > 4y - 11$

A word sentence may involve an unknown number or quantity without specifying a variable. In such cases, you may choose any variable to represent the number. Then write a mathematical sentence that relates the facts in the given situation.

EXAMPLE 2 Write a mathematical sentence that represents the given information.

a. The sum of a number and forty-nine is ninety-three.

b. The perimeter of a square is less than or equal to sixty centimeters.

SOLUTION **a.** Let n represent the unknown number.

$n + 49 = 93$

b. Let s represent the length of one *side* of the square.

$4s \leq 60$

Oral Exercises

Translate each word sentence into a mathematical sentence.

1. The sum of five and nineteen is twenty-four.
2. The difference when seven is subtracted from thirty-two is greater than twenty.
3. The product of a number a and three is less than or equal to forty-seven.
4. The total of five and a number b is thirty-nine.
5. The quotient when a number c is divided by six is two less than the product of c and six.
6. Fifteen less than twice a number d is greater than fifty more than one half d.

7. Thirty-five is not the product when the number that is one greater than a number e is multiplied by the number that is one less than e.
8. The quotient when the sum of a number f and twelve is divided by seven is sixteen more than f.
9. The cube of the difference when a number h is subtracted from a number g is greater than the sum of g and h.
10. The square of the sum of a number j and a number k is not equal to the sum of the square of j and the square of k.

Translate each mathematical sentence into a word sentence.

11. $r - 3 = 10$
12. $4s = 12$
13. $22 < t + 8$
14. $\frac{u}{5} \geq 5$
15. $7 - 2v \neq v^2$
16. $\frac{5}{w} = \frac{1}{2}w$
17. $8(9 - x) = 4$
18. $(6 + y)^3 \neq 6 + y^3$

Written Exercises

Write a mathematical sentence for each word sentence.

A

1. The total cost of x tickets at four dollars per ticket is seventy-six dollars.
2. After a deposit of y dollars in an account containing fifty dollars, the new balance is ninety-five dollars.
3. A customer used a twenty-dollar bill to pay for an item that cost z dollars and received less than four dollars in change.
4. A thousand-dollar profit was divided among w partners, and each partner received more than three hundred dollars.
5. The winner of the election received twenty-one votes, which is one more than two thirds of the total number, v, of votes cast.
6. In today's game the team scored nine runs, which is one less than twice the number of runs, r, that they scored in yesterday's game.
7. Mark is m years old, and his age in eight years will be three times his present age.
8. Donna is d years old, and her age one year ago was half her age six years from now.
9. The area of a rectangle that is six meters long and w meters wide is twenty-one square meters.
10. The perimeter of an equilateral triangle in which each side measures s centimeters is not fifty-four centimeters.
11. The total value of d dimes and q quarters is not ninety cents.
12. The total value of p pennies and n nickels is one dollar forty-six cents.

Choose a variable to represent the quantity described in parentheses. Then write a mathematical sentence that represents the given information.

13. Two more than twice a number is forty-two. (unknown number)
14. Eight less than one third a number is four. (unknown number)
15. One number is three times a second number, and their sum is thirty-two. (second number)
16. One number is five less than a second number, and their sum is greater than twenty-five. (first number)
17. When the cost of a new basketball is shared equally by five friends, each friend pays less than six dollars. (cost of the basketball)
18. When you receive a five-dollar discount on the original cost of a hair dryer, you pay only thirteen dollars. (original cost of the hair dryer)
19. The perimeter of a square is ninety-six centimeters. (length of one side of the square)
20. The area of a square is greater than one hundred square meters. (length of one side of the square)
21. The total value of a number of nickels is less than ninety cents. (number of nickels)
22. The total value of a number of quarters is four dollars twenty-five cents. (number of quarters)
23. The sum of a positive integer and the next greater positive integer is one hundred thirty-nine. (first integer)
24. The product of a positive integer and the next greater positive integer is six hundred fifty. (greater integer)

Choose a variable to represent one of the unknown quantities. Describe the quantity that the variable represents, then write a mathematical sentence that represents the given information.

B 25. When a strip of balsa wood that is fourteen centimeters long is cut into two pieces, one piece is four centimeters shorter than the other.
26. In a season of twenty-one games, the team won five times as many games as it lost, and one game ended in a tie.
27. The area of a rectangle whose length is three centimeters greater than its width is less than forty-five square centimeters.
28. The perimeter of a rectangle whose width is four meters less than twice its length is fifty-two meters.
29. The perimeter of a parallelogram in which one side is one centimeter shorter than one third the measure of the adjacent side is twenty-two centimeters.
30. The area of a triangle whose height is four times as great as the measure of its base is thirty-two square millimeters.
31. Half Jack's age twenty years from now will be the same as twice his age one year ago.

32. Eight years from now, Donna will be three times as old as she is now.
33. Carlos has five fewer dimes than quarters, and the total value of his dimes and quarters is $7.55.
34. If Leah had twelve more nickels, the total value of her nickels would be three times as great as the total value of the nickels she has now.

Choose a variable to represent one of the unknown quantities and write an open sentence that represents the given information. Then solve the open sentence over $\mathcal{R}$.

C 35. There are three more boys than girls in a class of twenty-five students.
36. Of twelve birds at the bird feeder, there are twice as many sparrows as robins.
37. When the product of five and a number is decreased by four, the result is the same as the result when the number is increased by eight.
38. When the quotient of a number divided by four is increased by five, the result is the same as the result when twice the number is decreased by nine.

Self-Test 3

VOCABULARY
mathematical sentence (p. 30)
equation (p. 30)
side of an equation (p. 30)
inequality (p. 30)
side of an inequality (p. 30)
open sentence (p. 31)
solution or root of an open sentence (p. 31)
solution set (p. 31)
satisfy (p. 31)
solve an open sentence (p. 31)
graph of an open sentence (p. 32)
set-builder notation (p. 32)

Solve each open sentence if $x \in$ {the positive real numbers} and graph its solution set.

1. $3x - 5 = 7$ 2. $x + 2 > 8$ 3. $x < x + 3$ *Obj. 1, p. 30*

Write a variable expression for each word phrase.

4. the quotient when the cube of the sum of a number m and one is divided by the sixth power of m *Obj. 2, p. 30*
5. the total value in cents of d dimes and x dollars

Write a mathematical sentence that represents the given information.

6. The square of the quotient when a number n is divided by three is less than the quotient when the square of n is divided by three. *Obj. 3, p. 30*
7. Bill is b years old now, and half his age in fourteen years will be the same as twice his age ten years ago.

Check your answers with those at the back of the book.

Chapter Summary

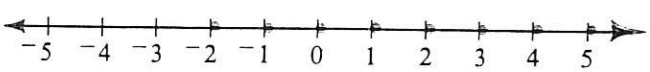
1. Any number that is either a positive number, a negative number, or zero is called a *real number*. The real numbers can be pictured as points on a *number line*, thereby showing their order. On a number line, the point that corresponds to a number is called the *graph of the number*, and the number that corresponds to a point is called the *coordinate of the point*.
2. A *set* is a collection of objects that are called the *members*, or *elements*, of the set. Some of the commonly used set symbols include { } (indicates a set), $\in$ (indicates that something is a member of a set), $=$ (indicates that two sets are equal), $\subset$ (indicates that one set is a subset of another), and $\emptyset$ (the *empty set*, or *null set*). If the members of two sets can be paired so that each member of each set is assigned to one and only one member of the other set, the relationship between the sets is said to be a *one-to-one correspondence*.
3. A *rational number* is any number that can be expressed as the quotient of two integers. All other real numbers are called *irrational numbers*.
4. A *numerical expression*, or *numeral*, is a name for a number. The number is called the *value of the expression*. You *simplify* a numerical expression when you replace it with the simplest, or most common, name of its value.
5. A *variable* is a symbol that is used to represent one or more numbers. The set of numbers that a variable may represent is called the *domain*, or *replacement set*, of the variable. An expression that contains a variable is called a *variable expression*. You *evaluate* a variable expression when you replace each variable with one of its values and simplify the resulting numerical expression.
6. When simplifying or evaluating an expression, *grouping symbols* indicate which operations are to be performed first. After the operations within grouping symbols have been performed, you continue with the *order of operations* outlined on page 25.
7. The product $7 \times 7 \times 7 \times 7$ may be written 7^4. In the expression 7^4, 7 is called the *base* and 4 is called the *exponent*.

8. An *equation* is a mathematical sentence which states that two expressions name the same number. An *inequality* is a mathematical sentence that is formed by placing an inequality symbol between two expressions. If a mathematical sentence contains at least one variable, it is called an *open sentence.* The set of all values of the variables for which an open sentence is true is called the *solution set* of the open sentence over the given domain.

Chapter Review

Write the letter of the correct answer.

1. What is the coordinate of the point on a number line that is five sixths of the way from $^-5$ to 7? 1-1
   a. $^-3$ b. $^-1$ c. 0 d. 5
2. Which of the following statements is false?
   a. $^-4 < 3$ b. $0 > {}^-9.5$ c. $^-7 > {}^-5 > 1$ d. $^-9 < {}^-2 < 0$
3. Let $A = \{0, 1\}$ and $B = \{0, 1\}$. Which of the following statements is false? 1-2
   a. $A \subset B$ b. $B \in A$ c. $\emptyset \subset A$ d. $0 \in B$
4. Let $S = \{x, y\}$. What is the total number of subsets of S?
   a. 1 b. 2 c. 3 d. 4
5. Which set of numbers is graphed on the number line below? 1-3



   a. {the integers greater than $^-3$}
   b. {the rational numbers greater than $^-3$}
   c. {the real numbers greater than or equal to $^-2$}
   d. {the integers between $^-3$ and 6}
6. Specify {the whole numbers between $^-30$ and 3} by roster.
   a. $\{^-29, {}^-28, {}^-27, {}^-26, \ldots, 2\}$ b. $\{^-29, {}^-28, {}^-27, {}^-26, \ldots, 0\}$
   c. $\{0, 1, 2\}$ d. $\{1, 2\}$
7. Simplify 2.5×3.24. 1-4
   a. 5.74 b. 3.49 c. 0.081 d. 8.1
8. Evaluate the expression $10 - \frac{j}{k}$ when $j = \frac{2}{3}$ and $k = \frac{1}{2}$.
   a. 7 b. $8\frac{2}{3}$ c. $9\frac{1}{3}$ d. $9\frac{2}{3}$

9. Simplify $\{6[24 - 6(3)] + 4\}2$. 1-5

a. 60 b. 80 c. 120 d. 656

10. Evaluate the expression $ab - \frac{a}{b}$ when $a = 1.5$ and $b = 3$.

a. 44.5 b. 43 c. 4 d. 2.5

11. Simplify $64 \div 4 + 4(10 - 2^2)$. 1-6

a. 40 b. 48 c. 264 d. 512

12. Evaluate the expression $n(n + 1)^2 - n^3$ when $n = 0.1$.

a. 0.0011 b. 0.003 c. 0.111 d. 0.12

13. Solve $z^2 = z + 2$ *if* $z \in$ {the whole numbers}. 1-7

a. $\{0\}$ b. $\{0, 2\}$ c. $\{2\}$ d. $\emptyset$

14. Solve $y \leq {}^{-}4$ if $y \in$ {the positive integers}.

a. $\{\ldots, {}^{-}8, {}^{-}7, {}^{-}6, {}^{-}5, {}^{-}4\}$ b. $\{{}^{-}4, {}^{-}3, {}^{-}2, {}^{-}1, 0, \ldots\}$

c. $\{{}^{-}4, {}^{-}3, {}^{-}2, {}^{-}1\}$ d. $\emptyset$

15. Translate the following word phrase into a variable expression: "the quotient when the sum of a number m and five is divided by the sum of five and the square of m" 1-8

a. $\frac{m + 5}{5 + m^2}$ b. $\frac{m + 5}{(5 + m)^2}$ c. $\frac{m + 5}{5 + 2m}$ d. $\frac{m + 5}{5^2 + m^2}$

16. Translate the following word sentence into an open sentence: "The sum of a positive odd integer n and the next greater positive odd integer is eighty-eight." 1-9

a. $n(n + 1) = 88$ b. $n(n + 2) = 88$

c. $n + (n + 1) = 88$ d. $n + (n + 2) = 88$

Chapter Test

1. On a number line, what is the coordinate of the point that is one fourth of the way from $^{-}3$ to 7? 1-1

2. List these numbers in order from least to greatest:

$4, {}^{-}2, {}^{-}1\frac{1}{2}, 3.25, 0, {}^{-}2.75$

Tell whether each statement is true or false.

3. $0 \notin$ {the positive real numbers} 1-2
4. $\{10, 20, 30, 40, 50\} \not\subset \emptyset$
5. {the natural numbers} = {the positive integers} 1-3
6. {the rational numbers} $\subset$ {the irrational numbers}

Evaluate each expression when $x = 4$ and $y = \frac{1}{2}$.

7. $5xy$ 8. $\frac{x}{y}$ 9. $\frac{x + y}{3}$ 1-4

Simplify.

10. $\{2[4 + 6(10)] \div 4\}5$ 11. $2\left\{\frac{6[16 - 2(4)]}{6 + 3(2)}\right\} - 8$ 1-5

12. $8 + (9 - 5)10 \div 2(7 + 3)$ 13. $3(5)^2 + (5 + 2^2)4 \div 2$ 1-6

Solve each open sentence if $z \in$ {the whole numbers}.

14. $\frac{z}{2} = 2z$ 15. $z < 3.2$ 16. $3z + 1 \geq 16$ 1-7

Write a variable expression for each word phrase.

17. the total number of hours in d days
18. the total value in cents of x dollars and q quarters 1-8

Write an open sentence for each word sentence.

19. The square of the sum of a number a and a number b is greater than the quotient when the product of a and b is divided by the sum of a and b. 1-9
20. Tina is t years old, and her age five years ago was one fourth her age sixteen years from now.

Contest Problems

The following are mathematical problems with a strong emphasis on logical reasoning. They are similar to problems that you might encounter in a mathematical contest or competition.

1. Jim knows that x is an integer greater than 2 and less than 8. Kim knows that x is an integer less than 9 and greater than 4. Tim knows that x is an integer greater than or equal to 2 and less than or equal to 6. How many possible values are there for x?
2. Starting with a certain number, Jane divided by 2 and then divided by 3. Her answer was $\frac{1}{15}$. However, she should have multiplied by 2 and then multiplied by 3. What is the correct answer?
3. Let a, b, and c be positive real numbers. If $ab = 48$, $bc = 96$, and $ac = 72$, what is the value of abc?
4. Let $x \in \{1, 2, 3, \ldots, 20\}$. For how many values of x will the value of the expression $\frac{3 + x}{5} + \frac{12}{x}$ be an integer?